A Weight Balanced Multi-Objective Topology Optimization for Automotive Development

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Abstract

Topology optimization in the field of automotive development strives for conceptual car components which are efficiently designed for multiple, partly conflicting loadings. Structures designed to manage nonlinear loading conditions which are often seen in crash events typically require a maximization of energy absorption while static loadings typically require a minimization of compliance of the structure. The hybrid cellular automata algorithm, which aims for a uniform distribution of the internal energy density, offers an efficient solution for the single objective problems but may fail where large energy differences occur in different load cases. Recently, it was demonstrated that a multi-objective optimization can be performed by linearly weighting and careful scaling of the load case energy levels. In this paper the approach is applied to the practical example of a vehicle control arm structure which is subject to two compliance load cases and an energy maximization load case and compared to a sequential optimization approach of the disciplines. Results demonstrate the practical feasibility of the proposed methods. Particularly, the scaled weighting approach yields a set of non-dominated trade-off solutions, facilitating the selection of a suitable balance between energy absorption and stiffness requirements.

1 Introduction

In the vehicle design process, a difficult challenge is to develop a lightweight structure which can meet both static stiffness requirements for vehicle performance targets such as NVH and ride handling as well as generating a structure that is efficient with regard to passive crash safety requirements. These requirements are often in conflict, and considering all requirements simultaneously has been a historical challenge. Such a multi-disciplinary design process [1] that involves different disciplines is often tackled not only using different software tools, but also requires several members of the design team working together. This can result in conflicting designs that must be resolved late in the design process potentially adding cost and weight to the overall structure.

The field of topology optimization refers to algorithms that aim at finding the optimum layout of a lightweight structure. In contrast to shape optimization, which targets fine-tuning of a rather advanced structural design, topology optimization provides the engineer or designer with a structural concept early in the design process. Concretely, topology optimization approaches address the problem of finding the optimal distribution of materials and void within a pre-defined design space [2]. For the defined load cases, topology optimization provides the geometric layout, including features such as holes of the structure defined by the design space. Topology optimization has been established as an efficient method to develop structures that are efficient for stiffness requirements, but the field of crashworthiness (i.e. energy absorption) remains a challenging application. The complexity due to the nonlinear nature of these crash type loads lends itself to heuristic approaches such as the hybrid cellular automata approach [3,4].

Recently, a methodology focusing on the concurrent optimization of load cases subject to stiffness and energy absorption objectives was proposed and tested on a simply supported beam example [5]. In this paper, the method is applied and evaluated on a practical model, specifically the model of a vehicle control arm. The structure developed by this concurrent optimization methodology is compared to a baseline approach of performing the static and dynamic optimization sequentially.

Section 2 describes the topology optimization method applied to the control arm. In Sec. 3, the model of the control arm, the design space, and its load cases are described. In Sec. 4 the parameters of the optimization is described and the results are presented and discussed. The paper is concluded in a summary in Sec. 5.

2 Weighted Crashworthiness Topology Optimization Method

This section presents the topology optimization approach to address both minimum compliance and maximum energy absorption in a concurrent multi-load case optimization. A brief introduction of topology optimization methods and targeted objective functions is presented as well.

Frequently applied for topology optimization are density-based methods [6]. These methods operate on a finite element mesh of the design space and assign a variable to each element. The originally binary variable that describes presence or absence of material within an element is replaced by a continuous density variable. Material properties of the element can be obtained based on the density values by using, for example, the popular Solid Isotropic Material with Penalization (SIMP) scheme [7]. In this case, a material property of the element, commonly the Young's modulus, is interpolated by a power law approach, according to

$$E_i(\rho_i) = \rho_i^p E_0 , \tag{1}$$

with density ρ_i with i=1,…N, the number of elements N and the Young's modulus of the original material E_0. To avoid intermediate densities these are penalized by a penalization exponent p.

A typical objective of linear static topology optimization is to maximize the stiffness of a structure, i.e. to minimize its compliance. The minimum compliance problem can be stated as:

$$\begin{aligned} &\min_{\vec{\rho}} c(\vec{\rho}) = \vec{u}^T \vec{f} \text{ subject to:} \\ &\qquad \vec{K}(\vec{\rho})\vec{u} = \vec{f} \\ &\qquad V(\vec{\rho}) = V_f \\ &\qquad 0 < \rho_{\min} \le \rho_i \le 1,\ i = 1,\dots,N, \end{aligned} \tag{2}$$

with the compliance c, the displacement vector $\vec{u}$ and the load vector $\vec{f}$. In the equilibrium condition, $\vec{K}$ is the stiffness matrix. A constraint V_f is imposed on the portion of the design space filled with material, i.e. the volume fraction $V(\vec{\rho})$ of the structure. In order to avoid numerical instabilities a minimum density $\rho_{\min}$ is defined.

In crashworthiness topology optimization, several other objectives are of importance such as avoiding intrusion into the cabin space, keeping acceleration curves smooth, or achieving high energy absorption. This work is focused on the latter, i.e. it is aimed for a maximization of the energy absorbed by the structure for the applied load. This can be formulated as:

$$\begin{aligned} &\max_{\vec{\rho}} EA(\vec{\rho}) \text{ subject to:} \\ &\qquad \vec{r}(\vec{\rho}) = 0 \\ &\qquad V(\vec{\rho}) = V_f \\ &\qquad 0 < \rho_{\min} \le \rho_i \le 1,\ i = 1,\dots,N, \end{aligned} \tag{3}$$

with the total energy absorbed by the structure $EA(\vec{\rho})$ and the residual $\vec{r}(\vec{\rho})$ of the finite element analysis.

For compliance problems, gradient-based methods based on analytical design variable sensitivities are commonly applied. However, due to the complexity of crashworthiness problems, analytical sensitivity information is difficult to obtain. Additionally, the nonlinearity of the problem might render a gradient descent problematic. This is why for crashworthiness topology optimization, usually heuristic methods are applied. A popular heuristic approach is referred to as Hybrid Cellular Automata (HCA). It was first devised as a method for the remodeling of bone growth, but it also provides efficient solutions for linear static problems [8, 9]. This heuristic approach was subsequently extended to nonlinear loading problems [3, 4].

The working principle of the HCA is the target of achieving a uniform distribution of a field variable throughout the structure by minimizing the deviation from a set-point. A new design is obtained iteratively by the update rule:

$$\rho_i^{new} = \rho_i + K_P\left(S_i - S^*\right), \tag{4}$$

where K_P is a scaling parameter, S_i is the field variable and S^* is the field variable set-point. Accordingly, when the change of the structure is approximating zero, the field variable is more uniformly distributed.
The HCA approach is able to consider several load cases in a multi-load case approach. In the multi-load case approach, a linear combination of the elemental field variables is applied:

$$S_i = \sum_{l=1}^{L} w_l S_{il} \ , \tag{5}$$

with weight w_l and field variable S_{il} referring to load case l and the number of load cases L.
The HCA approach has been demonstrated to yield efficient structures; however, the disciplines of crashworthiness and linear statics are typically considered in separate work. In [5] it is proposed to perform a multi-load case optimization in the multi-disciplinary case of concurrently dealing with a static and a nonlinear load case. This approach linearly weights the strain energy density as a field variable obtained from the static load case and the internal energy density as a field variable occurring in the nonlinear load case. Due to the large differences in energy levels for linear and nonlinear load cases, a scaling factor needs to be introduced such that the preference of the load case can be separated from the energy level:

$$S_i = \sum_{l=1}^{L} w_l S_{il} = \sum_{l=1}^{L} p_l \frac{S_{il}}{s_l} \, . \tag{6}$$

In this notation, the user's preference p_l for the load case is separated from the weight used in the optimization by refactoring the energy scale factor s_l. According to [5], the scaling factor is chosen such that the energy levels of all load cases are scaled down to the level of the load case with the lowest energy. For this purpose the energy of the full design space, i.e. ρ_i=1 for all elements is used:

$$s_l = \frac{\sum_{i=1}^{N} S_{il}^{full}}{\min_{l'} \sum_{i=1}^{N} S_{il'}^{full}} \tag{7}$$

The procedure of applying the method can be described as follows:

1. Define user preferences p_i for all load cases.
2. Compute scaling factor s_l from energy levels for all load cases, according to (7).
3. Compute weights w_l=p_l/s_l used for the linear combination of the field variables.
4. Run multi-load case HCA optimization.

A multi-objective study can be performed by applying several different preferences/weightings. Each of the load cases is treated as one separate optimization objective, such that a set of trade-off solutions in objective space is obtained.

3 Model of Vehicle Control Arm

In this work we consider the optimization of a vehicle control arm structure. In this section we describe the LS-Dyna model and the associated load cases. In general, the stiffness of the control arm is important for NVH, durability, and ride comfort and steering response; however, during a load retention scenario, it can be subject to substantial deformation which the structure must efficiently manage. Thus, during normal operation, minimum compliance is required, while during a loading event which results in yielding of the part, energy absorption should be maximized. We represent these requirements by two linear load cases (minimum compliance), and one nonlinear load case (maximize energy absorption).

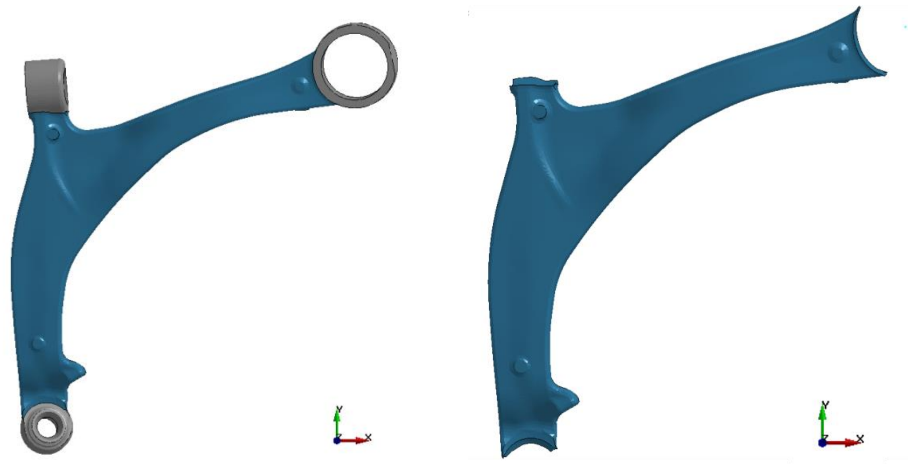

Fig.1: The LS-Dyna control arm model (left) and the design space for the optimization (right).

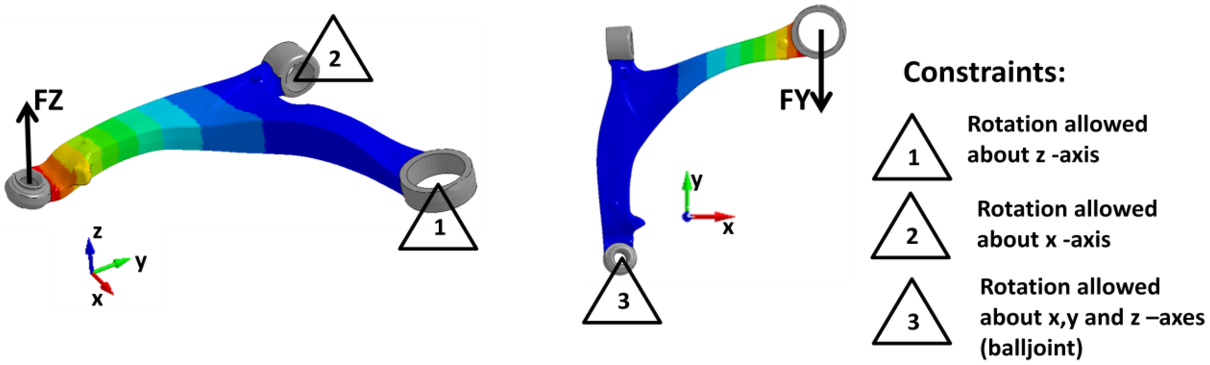

Fig.2: The LS-Dyna model of the static load cases with displacement contours shown.

Fig. 1 shows the geometry of the control arm represented as an LS-DYNA finite element model. The volume connecting the bushing mounts of the control arm is defined as the design space for the topology optimization. The volume of the entire control arm structure including the design space is meshed using tetrahedral elements with a mesh size of 3 mm nominal. The material of the control arm design space as well as the suspension and chassis at the ends are modelled with a piecewise linear elastic-plastic aluminium material model represented with `*MAT24`. This material is simplified to a `*MAT1` (equivalent Young's modulus and density) when the static load cases are solely considered. The control arm has a mass of 4.025kg of which the full density design space is 3.506kg.
There are two static load cases defined. The loads are applied as shown in Fig.2 directly to the control arm structure. The load cases are defined in z- and y-direction respectively, and are referred to in the remainder of this paper as load case FZ and load case FY. A static load of 10N is distributed to the ball joint mount point and bushing mount structure for the FZ and FY load cases respectively, and the constraints are defined as shown in Fig 2. These load cases represent normal operating conditions for which minimum compliance is the desired optimization objective. Care is taken that the linear assumption holds and the deformation stays within the linear region of the material model. There is one nonlinear load case defined (Fig. 3) where the control arm is subjected to a prescribed displacement in the x-direction which is increased from 0 to 110mm within 200ms. This model represents a laboratory test designed to ensure that the suspension components meet a minimum load requirement. This laboratory test is inspired by extreme loading conditions to the suspension system of the vehicle. A test fixture is mounted to the wheel hub, and a chain attaches the test fixture to a loading actuator which then applies the load. The relevant suspension components (damper, tie rod, knuckles) are modelled using a simplified beam representation in order to model realistic boundary conditions. The rubber bushings of the model are represented with solid elements using `*MAT_181_SIMPLIFIED_RUBBER/FOAM`. Although this controlled laboratory test is not inspired by any crash requirement, it provides a simple but ideal load case for the objective of maximizing energy absorption. The linear static load cases are analysed with LS-Dyna Implicit; the nonlinear load case is analysed with LS-Dyna Explicit.

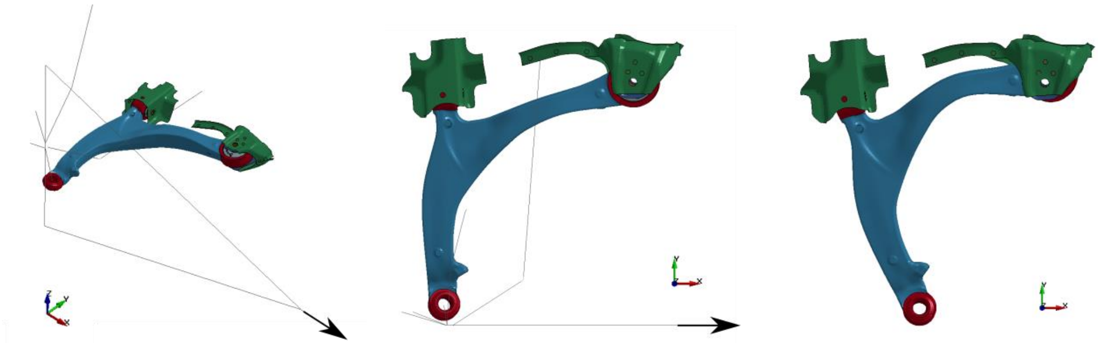

Fig.3: The LS-Dyna model of the nonlinear load case of the control arm. A prescribed displacement is applied to the chain. The rightmost figure shows the deformed structure when the load is applied to the full design space.

4 Experiments

The target of the work is to find a conceptual design that balances the different load cases, i.e., the objectives of minimum compliance and maximum energy absorption associated with the respective load requirements. This section describes the conducted experiments and discusses achievement of the targeted objectives.
In order to obtain baselines for comparison, each objective was considered separately. Secondly, a sequential approach is applied based on the idea of first optimizing the compliance objectives, followed by a second optimization for the energy absorbing targets. Finally, we apply the multi-objective approach based on scaled weighting. All experiments implement the software package LS-TaSC [10,11], the topology optimization software designed to work with LS-DYNA. LS-TaSC provides a straightforward interface to implement the HCA algorithm described in Sec. 2, including the capability to weight multiple load cases. Unless stated otherwise, all optimization runs converged with respect to mass redistribution. The mass target 1,718 kg of the design space, corresponding to a volume fraction of V_f=0.49. The neighbourhood radius was defined at 9mm based on the mesh size of 3mm in the design space. The move limit was reduced from a default value of 0.1 to 0.02 in order to allow the optimization algorithm to evolve the structure without generating instabilities. The LS-TaSC parameters are summarized in Tab. 1. All results are evaluated as obtained from the optimization, post-processing is not considered within the scope of this paper. The section is concluded with a discussion on the achieved objective values in Sec. 4.4.

HCA Parameter (LS-TaSC Setting)	Parameter Value
Mass Fraction	0.49
Minimum Length Scale (Neighbor Radius)	9mm
Move Limit	0.02
Convergence tolerance	0.002

Table 1: LS-TaSC parameter settings.

4.1 Separate Optimization of Disciplines

Initially, the static and the nonlinear load cases were run separately in order to understand the resulting structures and obtain solutions that represent the practical optimum that can be achieved with the applied optimization tool.
The scaling factor for the static load cases is determined according to (6) by measuring the overall internal energy of the design space as in the LS-Dyna **MATSUM** database. Although the scaling method is proposed for combining nonlinear and static loads, it can of course be used as well to consistently obtain appropriate scaling factors and weights for pure static multi-load case topology

optimization. Since both load cases are considered of equal importance, the same preference is chosen for both, so that neither load case dominates the structure. The internal energy of the FZ load case is approximately 2.2 times that of the FY load case; therefore, a weight of 0.69 was applied for the FY load case and a weight of 0.31 was applied for the FZ load case The optimization objective for the static load cases is compliance, which is measured by the stroke at the load point.

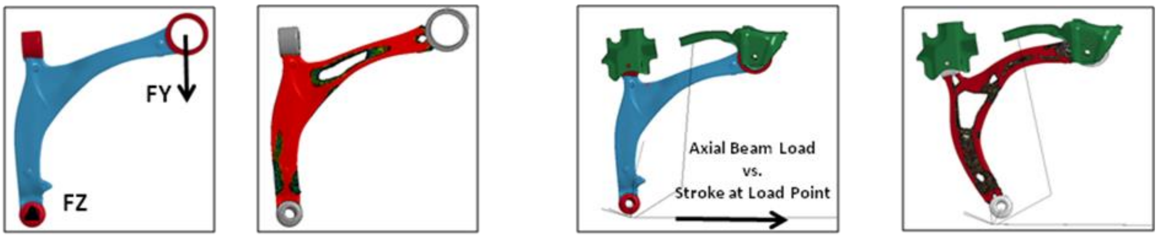

Fig.4: Loading and solution resulting from multi-load case optimization of the static load cases (left) and loading and solution resulting from single-load case optimization of the nonlinear load case (right).

The nonlinear load case is applied to the same design space that is defined for the static load cases. For this loading, rather than minimum compliance, the desired outcome is maximizing energy absorption for the whole structure under this loading. Energy absorption was calculated by measuring the force in the chain and integrating it over the stroke. LS-TaSC was run for both disciplines separately. The resulting structures are shown in Fig. 4.
A leading motivation for this study is that often the structures that are optimized for stiffness load cases perform very poorly for energy absorption and vice versa. This is a significant challenge for applications in which both requirements are important design criteria. In order to demonstrate this issue, the optimized structure that was generated for the compliance load case is also evaluated with respect to the nonlinear load case and vice versa. The results are discussed in Sec. 4.4.

4.2 Sequential Optimization of Disciplines

As an initial attempt to combine linear and nonlinear load requirements (compliance and energy absorption), a sequential approach is used. The entire design space is optimized subject to the static load cases for minimum compliance, as described in Sec. 4.1 but with a reduced mass fraction of 0.3. The resulting design was then removed from the design space. A mass fraction of 0.30 is then assigned to the remaining structure for the optimization of the dynamic load case, such that the overall mass target of 1.718 kg is achieved. The optimization process and the resulting structure are shown in Fig. 5. The deformation of the structure subject to the dynamic load is shown in Fig. 8 (left).

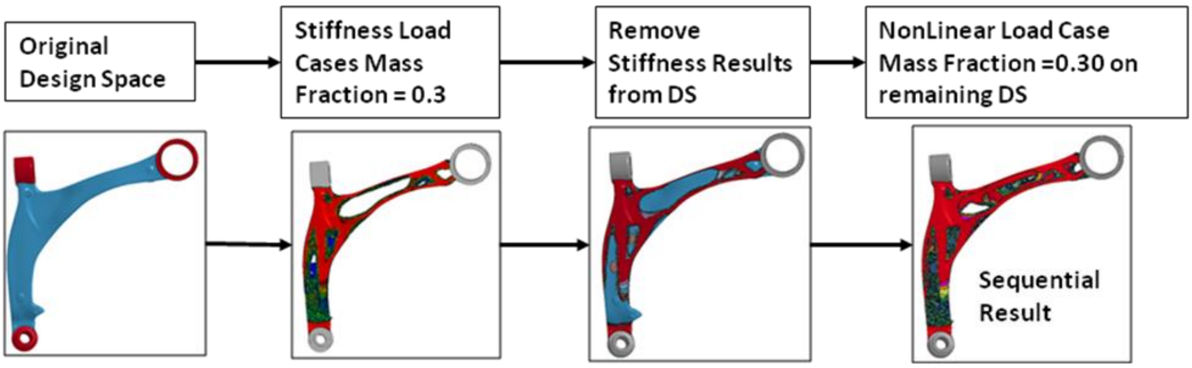

Fig.5: Sequential approach to combine linear (minimize compliance) and nonlinear (maximize energy absorption) load cases for a single structure.

4.3 Multi-objective Optimization of Disciplines Using Scaled Weighting

Although the sequential process addresses both stiffness and energy absorption as objectives, there is little control as to the balance of the load cases. With the target to balance the disciplines of dynamic and static loads, a multi-objective optimization approach is applied following the approach described in Sec. 2. All load cases are optimized concurrently, based on user defined preferences. From the energy levels of the load cases and the defined preferences, suitable weights are computed.

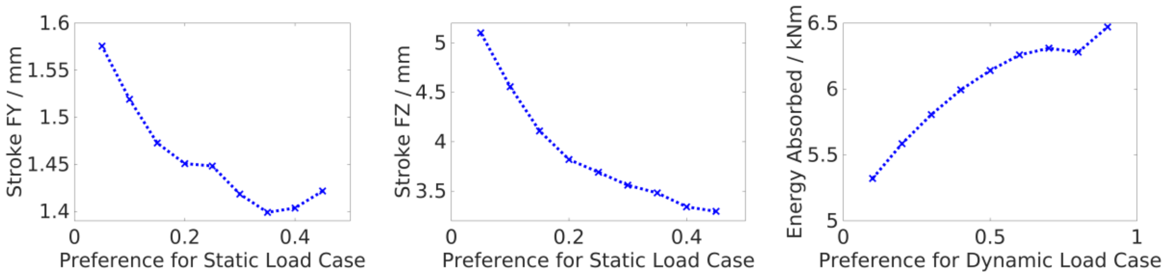

Fig.6: Dependence of the objective value on the preference for the static FY (left), the static FZ load (mid) and the nonlinear load case (right).

In order to obtain a set of trade-off solutions that will balance the design to different degrees, different sets of preferences are defined. The preference for the dynamic load is chosen as:

$$p_{dyn} = [0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9].$$

The static load cases preferences are computed based on the dynamic load case preference:

$$p_{FY} = p_{FZ} = (1 - p_{dyn})/2.$$

These are chosen equally since no static load case is preferred over the other. From the preferences, the weights for the optimization can be determined using the scaling factors computed from the internal energy. This data is obtained from analyzing the structure with the complete design space filled with material.
In practice, topology optimization analyzed with LS-TaSC requires a consistency of the design space. For this reason, geometry, elements, and element numbering of the design space need to be exactly consistent for the different load cases. An optimization study is conducted such that a topology optimization result is generated for each preference value. Accordingly, a study consisting of nine different optimizations is conducted.

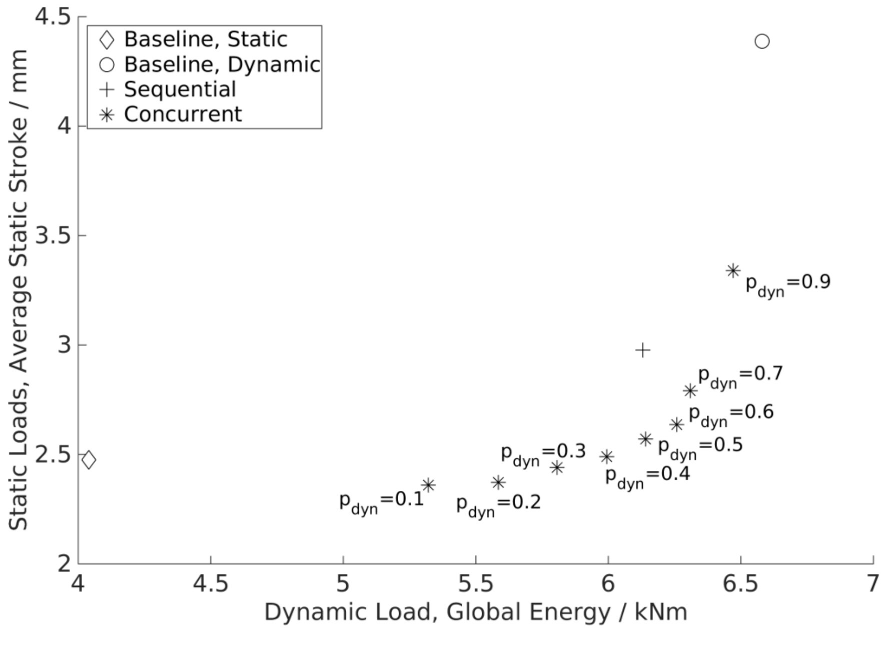

Fig.7: The objective values of the structures obtained from running the concurrent optimization of nonlinear and linear loads for different user preferences. Also results from Sec. 4.1 and Sec 4.2 are shown. The two linear load cases objectives are combined by averaging.

Fig.8: Resulting deformed structures of trade-off from running the sequential topology optimization of stiffness and energy absorption (left), and concurrent scaled weighting topology optimization, resulting from an equal preference of compliance and energy absorption (right).

The expectation is that for increasing the preference of a particular load case, the respective objective value shows a clear trend of improvement. In fact, the stroke of the static load cases is decreasing for the static load cases and the energy absorption is increased for the dynamic load case when the respective preference is increased. This can be observed in the plots of the resulting preference dependencies in Fig. 6. Possibly due to the heuristic nature of the HCA approach as well as the high number of variables, the observed trend is not ideal. Yet, the proposed approach shows a clearly defined regularization of the objective values based on the preferences.
Ideally, the optimization study would result in nine non-dominated trade-off solutions uniformly distributed on the Pareto front. A non-dominated solution is a solution that is, when compared to any of the other solutions, better in at least one of the objectives. Since three load cases are considered, the user is confronted with solutions distributed in a three dimensional objective space. Choosing the "best" solution from such a set is, in itself, a non-trivial task, since with respect to the objective values, none of the non-dominated solutions is better than another.
In this work, we are mainly interested in balancing the disciplines of energy absorption and compliance. Therefore, in order to facilitate the task of choosing one trade-off solution, the strokes, as measure of compliance, are combined to one objective by averaging. The solutions in the resulting two dimensional objective space are shown in Fig. 7. Note that one dominated solution is omitted in the plot, so that a set of eight non-dominated trade-off solutions remains. The approach provides the user with the freedom to choose the solution with the most appropriate balance for the design application. For further analysis in this work, we simply choose the solution for which the preferences are equally distributed between the dynamic and the static load cases, i.e. the result for p_{dyn}=0.5 is chosen as representative trade-off. The resulting structure is shown in Fig. 8 (right).

4.4 Discussion

Tab. 2 and Fig. 9 present the results of the different experiments. All values are normalized with respect to mass to alleviate small differences in the achievement of the mass target. Intuitively, optimization of the separate disciplines is expected to yield the best result in their respective objective value. Thus these results are considered as a practical "Baseline Result" and assigned 100% performance.
As described in Sec. 4.1, the baseline structures were also evaluated for the opposing load cases, i.e. the optimized structure that was generated for the compliance load case is also evaluated with respect to the nonlinear load case and vice versa. The performance is shown designated as "Opposing Result". It demonstrates that applying a structure optimized for the compliance load cases performs poorly for energy absorption load case as well as a structure optimized for the energy absorption load case performs poorly for the compliance load cases.
Aiming for a balance of the load cases, an approach implementing a sequential design optimization process was applied as described in Sec. 4.2. This is an intuitive, practical approach, also motivated by the fact that it can account for the use of different tools for different optimization objectives. However, there are inherent disadvantages to this approach including giving a preference for the first

load objective, and lack of control other than the initial optimization parameters as to the balance between the load cases.
The result of the sequential approach is shown in Fig. 9 and Tab. 2 as “Sequential Result”. In this case, the static FY and the dynamic load case perform nearly as well as the structures developed exclusively for either the static or nonlinear load cases. The reduction of the design space for the nonlinear optimization did not have a too severe impact on the performance of the resulting structure. However, degradation is noted for the static FZ load case. This approach does show an overall improvement in the balance of all load cases over the designs that were developed without considering the conflicting loads. Yet, there is little control on balancing the load cases. If a different solution is desired with regards to the balance between the conflicting objectives, the entire process must be repeated with adjusted mass fractions that are at best an educated guess for each load case type.
This motivates the approach to concurrently optimize all load cases. The trade-off solution can be influenced by a user preference which is mostly decoupled from the energy levels of the load cases by using the proposed scaling. Systematically, several runs can be performed to obtain a set of trade-off structures from which the user can choose according to the application. The chosen trade-off for this case is that of the optimization that assigns equal preferences between static and nonlinear objectives. It is shown in Fig. 9 and Tab. 2 as “Concurrent Result”. Although the structure is not very different from the result of the sequential optimization (Fig. 8), it shows better objective values. Of particular note is that the FY loading case for the concurrent optimization outperforms the baseline result which consider the static loads separately. This is possibly due to similar optimal solutions for the dynamic load case and the FY load case which effectively increases the weight preference of the FY case over that of the baseline result. Overall, the concurrent solution is the best *balanced* structure found in terms of achieving all the objectives defined for this structure.

Optimization Run	**Static FZ**		**Static FY**		**Dynamic**	
	[mm^{-1} / kg]	**[%]**	**[mm^{-1} / kg]**	**[%]**	**[kNm / kg]**	**[%]**
Baseline Result	0.17	**100**	0.66	**100**	3.83	**100**
Opposing Result	0.08	**49**	0.56	**84**	2.36	**62**
Sequential Result	0.13	**78**	0.64	**97**	3.58	**94**
Concurrent Result	0.16	**94**	0.69	**106**	3.63	**95**

Table 2: Optimization results.

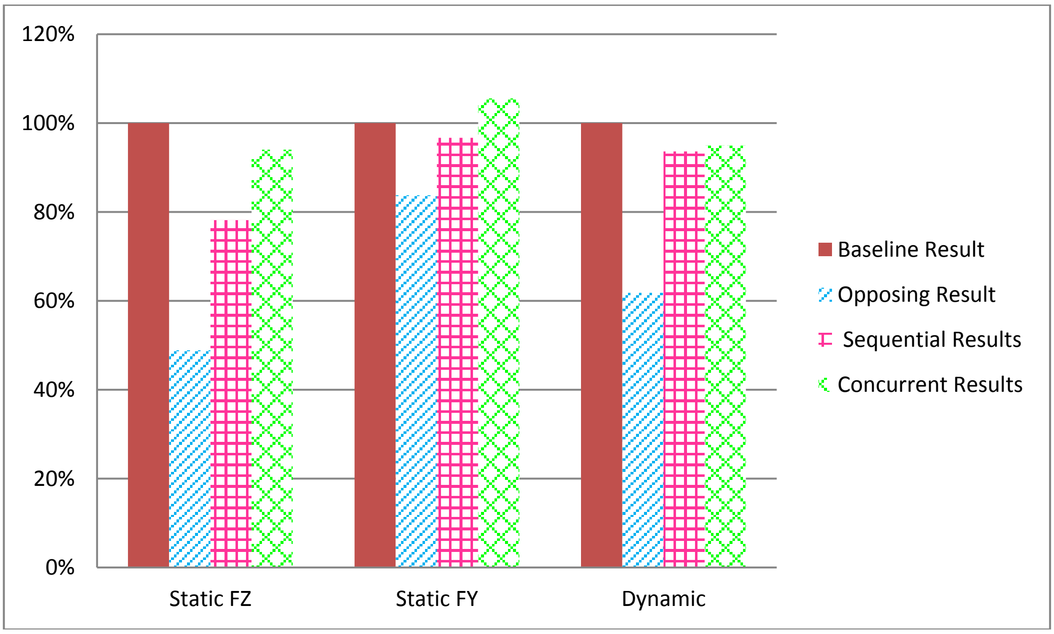

Fig.9: Summary of topology optimized control arm structure objectives normalized to baseline.

For further more practical evaluation of the results, there is a need for post-processing, which is not considered within the scope of this paper. Here, structures that still contain a small amount of intermediate densities are compared. The next step is a post-processing of the results to a structure consisting only of material and void. This is another non-trivial problem especially for the dynamic load case. The mass fraction as well as the deformation mode must be maintained requiring the manual involvement of an engineer, and is a possible direction for future research.

5 Summary

In this paper a topology optimization with regard to static and nonlinear requirements was conducted. The hybrid cellular automata algorithm was applied for topology optimization of both objectives. Experiments were performed on a practical LS-Dyna model of a vehicle control arm. The results confirm the suitability of the applied algorithm for the considered optimization objectives. Two approaches for balancing the multi-disciplinary requirements were evaluated. The first trade-off structure is obtained by sequentially optimizing static and dynamic load cases. Another structure is obtained by a multi-objective study, performing a topology optimization of static and dynamic load cases concurrently, using a scaled weighting approach for the energy aggregation. Results show that both methods are suitable to obtain feasible trade-off solutions, yet the scaled weighting approach provides a better solution and the possibility for the engineer to specify preferences that provide more control on balancing the objectives.

6 Acknowledgements

We thank Willem Roux from LSTC, for support, especially with regard to the implementation of the variable move limit functionality for LS-TaSC.

7 Literature

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