Corpuscular method for OOP simulations, difficulty dealing with small vent holes

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Summary:

The corpuscular method for airbag deployment simulations in LS-DYNA has quickly gained popularity. The method is inherently stable, easy to use and relatively fast. One drawback, though, is poor handling of small vent holes. The gas flow is underestimated in cases where the particle mean free path is of the same order of magnitude as the vent hole size. In such situations the local flow pattern can not be resolved and the gas has a local behaviour that resembles the one of free molecular flow. This article simply compares the venting as predicted by Wang-Nefske with the venting obtained at free molecular flow. This gives a rough estimate of the worst possible performance of the corpuscular method in the treatment of venting.

Keywords:

Corpuscular method, venting, Wang-Nefske

1 Introduction

Following the kinetic molecular theory, cf. Maxwell [1], the mean free path at thermal equilibrium is

$$l = \frac{V}{\sqrt{2}\pi N r_p^2}$$

Here V is the volume, N is the number of particles and r_p is the particle radius. As r_p is defined in LS-DYNA the mean free path at 10,000 particles per liter is roughly 1cm. This corresponds to a computationally realistic case with 1,000,000 particles inside a 100 liter bag. Considering vent holes with a diameter of few cm it is fairly obvious that the local flow pattern will not match reality. The mean free path is simply too large and the local gas behavior will resemble what one has at free molecular flow.

To get a rough estimate of the error in the predicted mass flux, the venting at free molecular flow can be compared to an analytical expression that is used in the control volume approach to airbag modeling.

2 Venting according to Wang-Nefske

According to Wang et.al [2] the mass flow rate through a vent hole with area A is

$$\dot{m}_{w-n} = \frac{MAp}{R\sqrt{T}} Q^{1/\gamma} \sqrt{\frac{2\gamma R}{M(\gamma-1)}\left(1 - Q^{\gamma-1/\gamma}\right)} \tag{1}$$

Here R is the universal gas constant, M is the molar mass, $\gamma = C_p/C_v$ is the ratio of heat capacities and p and T are the pressure and temperature inside the bag, respectively. Q is a dimensionless function of the ratio between the pressure outside and inside the bag. The expression is very accurate at a stationary gas flow.

To make things simple we assume choked flow where

$$Q = Q_c = \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}} \tag{2}$$

The ideal gas law states that

$$p = \frac{\rho R T}{M} \tag{3}$$

From Equations (1)-(3) the mass flow rate can be rewritten as

$$\dot{m}_{w-n} = \rho A \sqrt{\frac{RT}{M}} \sqrt{\gamma \left(\frac{2}{\gamma+1}\right)^{\gamma+1/\gamma-1}} \tag{4}$$

3 Free molecular flow

Having holes with a diameter of the same order of magnitude as the particle mean free path the local flow pattern can not be resolved. The leakage rate will be close to what one obtains at free molecular flow. That is, the leakage one would have if there is no interaction between the particles in the gas.

$$\dot{m}_{free} = m_p f_c \tag{5}$$

m_p is he particle mass and f_c is the frequency at which particles are passing the vent hole. This frequency can be derived directly from the Maxwell-Boltzmann velocity distribution, cf. Maxwell [1].

$$f_c = \frac{NAv_{rms}}{\sqrt{6\pi}V} \tag{6}$$

Here v_{rms} is particle the root-mean-square velocity, cf. Olovsson [3].

$$v_{rms} = \sqrt{\frac{3RT}{M}} \tag{7}$$

Following Equations (5)-(7) the mass flow rate at free molecular flow can be expressed as

$$\dot{m}_{free} = \frac{m_p N}{V} A\sqrt{\frac{RT}{M}}\frac{1}{\sqrt{2\pi}} = \rho A\sqrt{\frac{RT}{M}}\frac{1}{\sqrt{2\pi}} \tag{8}$$

From Equations (4) and (8) the ratio between the mass flow rate predicted by Wang-Nefske and the one obtained at free molecular flow is

$$\frac{\dot{m}_{free}}{\dot{m}_{w-n}} = \frac{1}{\sqrt{2\pi\gamma\left(\frac{2}{\gamma+1}\right)^{\gamma+1/\gamma-1}}} \tag{9}$$

As γ can range between 1 and 5/3, this ratio stays between 0.55 and 0.66. Hence, having too few particles the estimated leakage at choked flow can be underestimated by more than 40%. For flow that is not choked the error depends on how the exterior air is defined to influence the particle flow.

4 Summary

Poor resolution of the flow pattern near small vent holes is an obvious weakness of the corpuscular method. A rough estimate shows that the mass flow rate at choked flow, in worst cases, can be underestimated by more than 40%.

However, it is to be mentioned that a feature that detects and corrects for inadequate venting is under development. The idea is too look at the ratio between mean free path and vent hole size and to make a local correction to the particle flow pattern.

5 Literature

[1] Maxwell, J.C.: “Illustrations of the Dynamical Theory of Gases”, Philosophical Magazine, vol. 19, 1860, pp. 19-32.

[2] Wang J.T. and Nefske O.J.: "A new CAL3D Airbag Inflation Model", SAE paper 880654, 1988.

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