Fluid - Structure - Interaction - A Still Challenging Topic

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Fluid structure interaction (FSI) problems are of great relevance in many engineering fields. Profound understanding of fluid structure interation is essential to explain and predict a wide range of physical phenomena among which are fluid sloshing in tanks due to horizontal wind forces or earthquake, wind-induced vibration of slender bridges or high-rise buildings, vibrating pipes and the dynamics of offshore structures due to cyclic sea currents.

This talk focuses on a wide spread FSI-subclass which studies the behaviour of incompressible viscous flows and thin-walled structures exhibiting large deformations. Free surfaces often present additional challenges for such problems. In order to understand fluid structure interaction and to obtain reliable numerical results both the structural and the fluid part have to be adaequately modelled and properly coupled.

The equations governing the first field, the fluid velocity and pressure, are naturally written in an Eulerian (spatial) framework. Thus the fluid is described in a spatial coordinate system. The structure field, on the other hand, is most appropriately formulated in a Lagrangean coordinate system which follows the material displacement.

In order to close the gap between these two descriptions an Arbitrary Lagrangean Eulerian (ALE) formulation is applied for the fluid field. This allows for solving the fluid equations on an arbitrarily moving grid. Furthermore free fluid surfaces can be included using an ALE framework. The ALE formulation however requires the solution for the fluid mesh which is treated as the third field within the coupled problem.

In order to ease coupling Finite Element approximations are chosen for all participating fields.

The fluid is modelled as an incompressible viscous Newtonian fluid. Its behavior is described by the incompressible Navier-Stokes equations on the temporarily varying fluid domain Ω_f. These equations formulated for the unknown fields of velocity and pressure read

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} - \mathbf{u}^G) \cdot \nabla \mathbf{u} - 2\nu \nabla \cdot \varepsilon(\mathbf{u}) + \nabla p = \mathbf{f} \;\; \text{in } \Omega_f \times (0,T)$$

$$\nabla \cdot \mathbf{u} = 0 \quad \text{in} \quad \Omega_f \times (0,T),$$

with appropriate initial and boundary conditions. Here $\mathbf{u}$ denotes the unknown fluid velocity, p the unknown kinematic pressure, $\mathbf{u}^G$ the grid velocity, ν the kinematic viscosity, $\mathbf{f}$ the body force in the fluid, $\varepsilon(\mathbf{u}) = \frac{1}{2}[\nabla \mathbf{u} + (\nabla \mathbf{u}^T)]$ the rate of deformation tensor and the stress tensor is defined by $\sigma = -p\mathbf{I} + 2\nu\,\varepsilon(\mathbf{u})$. Free surface effects are incorporated via a local Lagrange or height function approach.

The classical Galerkin FEM formulation of the incompressible Navier Stokes equations exhibits a number of numerical difficulties and problems and requires additional stabilization means. A Galerkin least-squares formulation is used in order to stabilize the variational form.

The structural domain is described by the equations of geometrically nonlinear elastodynamics

$$\rho \ddot{\mathbf{d}} = \nabla \cdot \mathbf{S} + \rho \mathbf{b} \quad \text{in} \quad \Omega_s \times (0,T)$$

with appropriate initial and boundary conditions.

The fluid mesh is determined such that it sticks to the moving structural surface as well as rigid walls while keeping the mesh deformation small and retaining admissible element distortions. An example of a mesh, deforming with an rapidly changing fluid domain is depicted in figure 1 a). The mesh topology is kept while the single nodes move. Intelligent mesh moving strategies reduce the need for remeshing significantly.

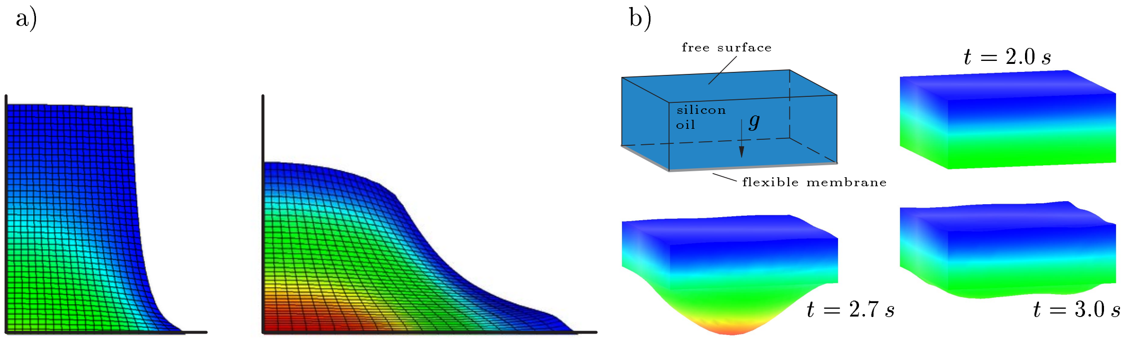

Figure 1: a) Collapsing water column with increasingly deforming ALE mesh b) Tank with flexible membrane bottom (pressure solution)

Fluid-structure interaction is a classical surface coupled multi field problem. Both fields influence each other along the common interface where information has to be exchanged during the simulation. It proves advantageous to solve the single fields sequentially. Hence a partitioned iterative staggered algorithm is used to solve the coupled system in every time step. Subiterations over the fields ensure continuity of displacements and forces along the coupling fluid-structure interface and guarantee a stable and accurate numerical simulation even over long time intervals.

The talk will include a short remark on turbulence modelling where an Large Eddy simulation (LES) based on the variational multiscale method (VMS) is stressed.

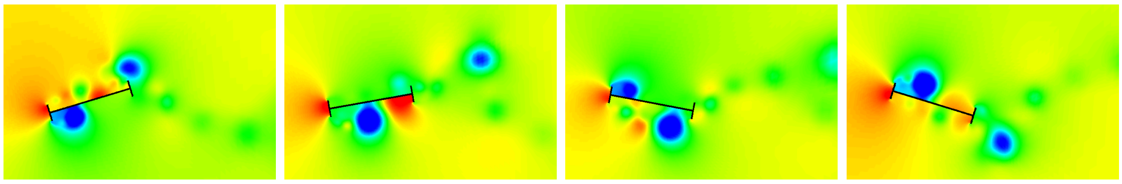

Figure 2: Bridge cross section subjected to wind induced vibrations (pressure solution)

A number of examples will be presented. Figure 2 shows the result of a long-time simulation. The depicted bridge cross section has been subjected to horizontally moving fluid. The cross section is sensitive to vortex shedding which causes increasing vibration. This eventually leads to failure of the structure.

An example for a 3D-simulation of a filled liquid storage tank with rigid walls and a flexible membrane at its bottom is depicted in figure 1 b). For the numerical computation the system is kept in equilibrium until $t = 2.1\,s$ by a surface load at the bottom. Then the load is removed and the system starts to oscillate exhibiting large deformations.

References

[1] W.A. Wall, *Fluid-Struktur-Interaktion mit stabilisierten Finiten Elementen.*, Ph.D.-Dissertation, Report No. 31 (1999), Institute of Structural Mechanics, University of Stuttgart.

[2] D.P. Mok, *Partitionierte Lösungsansätze in der Strukturdynamik und der Fluid-Struktur-Interaktion.* Ph.D.-Dissertation, Report No. 36 (2001), Institute of Structural Mechanics, University of Stuttgart.

[3] V. Gravemeier, W.A. Wall and E. Ramm, *A three-level finite element method for the instationary incompressible Navier-Stokes equations.*, Computer Methods in Applied Mechanics and Engineering 193 (2004) 1323-1366.

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Bamberg, Germany 14.–15. October 2004

Fluid Structure Interaction – Coupling, Free Surface, Turbulence

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Incompresible flows & thin–walled structures

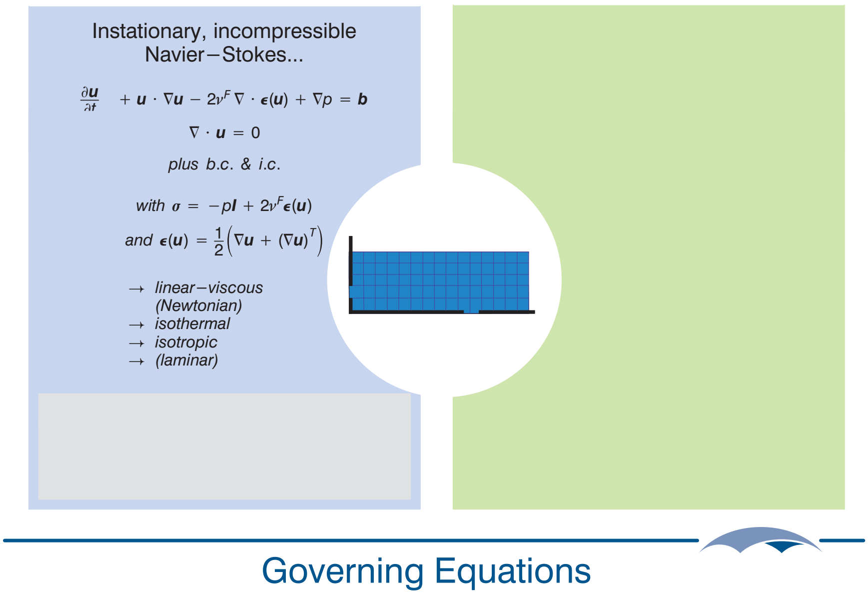
Fluid Structure Interaction

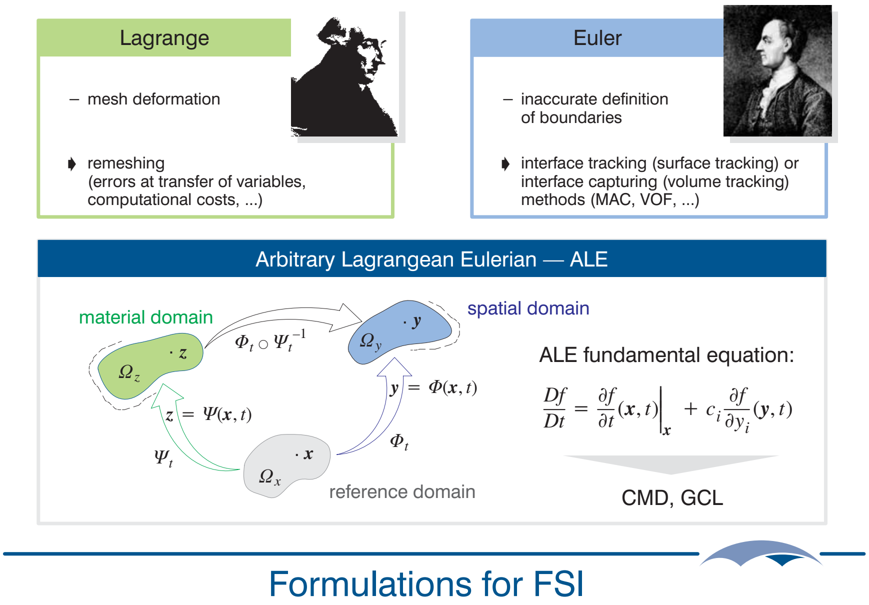
- Formulation
- Coupling
- Free Surface
- Turbulence
- Conclusions

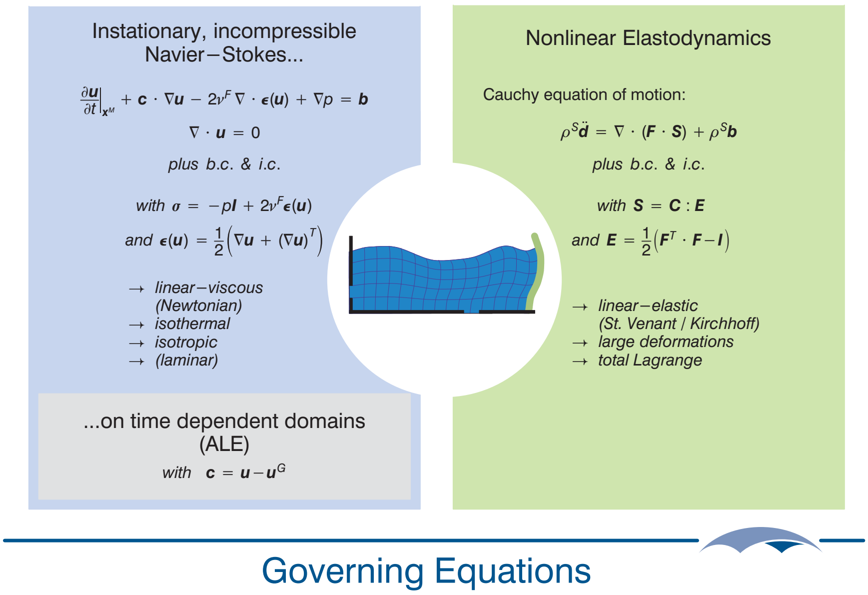
Outline

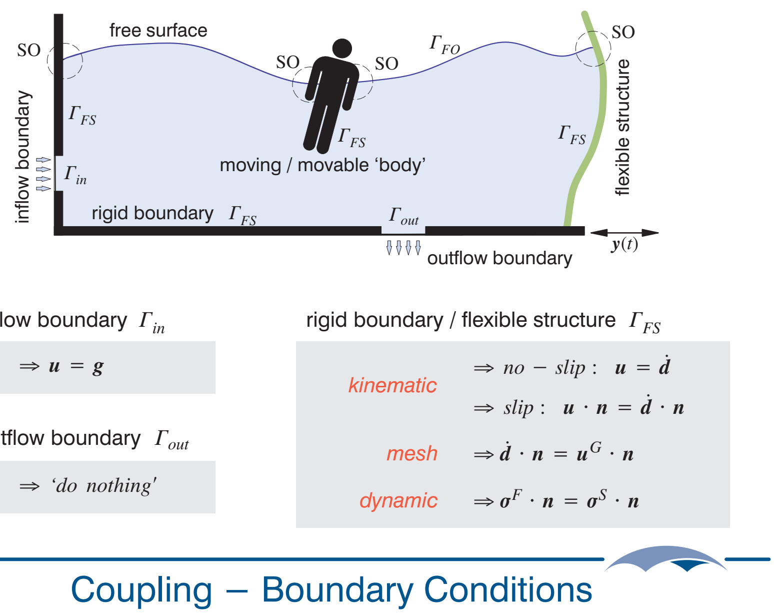
- Formulation
- Coupling
- Free Surface
- Turbulence
- Conclusions

Outline

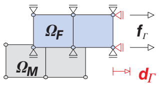

Instationary, incompressible Navier–Stokes...
$\frac{\partial \boldsymbol{u}}{\partial t} + \boldsymbol{u} \cdot \nabla \boldsymbol{u} - 2\nu^F \nabla \cdot \boldsymbol{\epsilon}(\boldsymbol{u}) + \nabla p = \boldsymbol{b}$
$\nabla \cdot \boldsymbol{u} = 0$
plus b.c. & i.c.
with $\sigma = -p\boldsymbol{I} + 2\nu^F \boldsymbol{\epsilon}(\boldsymbol{u})$
and $\boldsymbol{\epsilon}(\boldsymbol{u}) = \frac{1}{2}\left(\nabla \boldsymbol{u} + (\nabla \boldsymbol{u})^T\right)$
→ linear–viscous (Newtonian)
→ isothermal
→ isotropic
→ (laminar)
Governing Equations

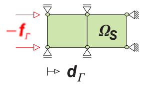

Lagrange
– mesh deformation
remeshing (errors at transfer of variables, computational costs, ...)
Euler
– inaccurate definition of boundaries
interface tracking (surface tracking) or interface capturing (volume tracking) methods (MAC, VOF, ...)
Arbitrary Lagrangean Eulerian — ALE
material domain
spatial domain
reference domain
$\Phi_t \circ \Psi_t^{-1}$
Ω_z
Ω_y
Ω_x
$z = \Psi(x, t)$
$y = \Phi(x, t)$
Ψ_t
Φ_t
ALE fundamental equation:
$\frac{Df}{Dt} = \frac{\partial f}{\partial t}(x, t)\Big|_x + c_i \frac{\partial f}{\partial y_i}(y, t)$
CMD, GCL
Formulations for FSI


Instationary, incompressible Navier–Stokes...
$\frac{\partial \mathbf{u}}{\partial t}\Big|_{x^M} + \mathbf{c} \cdot \nabla \mathbf{u} - 2\nu^F \nabla \cdot \epsilon(\mathbf{u}) + \nabla p = \mathbf{b}$
$\nabla \cdot \mathbf{u} = 0$
plus b.c. & i.c.
with $\sigma = -p\mathbf{I} + 2\nu^F \epsilon(\mathbf{u})$
and $\epsilon(\mathbf{u}) = \frac{1}{2}\left(\nabla \mathbf{u} + (\nabla \mathbf{u})^T\right)$
→ linear–viscous (Newtonian)
→ isothermal
→ isotropic
→ (laminar)
...on time dependent domains (ALE)
with $\mathbf{c} = \mathbf{u} - \mathbf{u}^G$
Nonlinear Elastodynamics
Cauchy equation of motion:
$\rho^S \ddot{\mathbf{d}} = \nabla \cdot (\mathbf{F} \cdot \mathbf{S}) + \rho^S \mathbf{b}$
plus b.c. & i.c.
with $\mathbf{S} = \mathbf{C} : \mathbf{E}$
and $\mathbf{E} = \frac{1}{2}\left(\mathbf{F}^T \cdot \mathbf{F} - \mathbf{I}\right)$
→ linear–elastic (St. Venant / Kirchhoff)
→ large deformations
→ total Lagrange
Governing Equations


free surface
SO
SO
SO
SO
Γ_{FO}
Γ_{FS}
Γ_{in}
inflow boundary
moving / movable 'body'
flexible structure
rigid boundary Γ_{FS}
Γ_{out}
outflow boundary
$y(t)$
inflow boundary Γ_{in}
$\Rightarrow u = g$
outflow boundary Γ_{out}
$\Rightarrow$ 'do nothing'
rigid boundary / flexible structure Γ_{FS}
kinematic
$\Rightarrow$ no – slip : $u = \dot{d}$
$\Rightarrow$ slip : $u \cdot n = \dot{d} \cdot n$
mesh
$\Rightarrow \dot{d} \cdot n = u^G \cdot n$
dynamic
$\Rightarrow \sigma^F \cdot n = \sigma^S \cdot n$
Coupling – Boundary Conditions

FLUID
MESH





STRUCTURE

Instationary, incompressible Navier–Stokes...

$$\left.\frac{\partial \mathbf{u}}{\partial t}\right|_{\mathbf{x}^M} + \mathbf{c} \cdot \nabla \mathbf{u} - 2\nu^F \nabla \cdot \boldsymbol{\epsilon}(\mathbf{u}) + \nabla p = \mathbf{b}$$

$$\nabla \cdot \mathbf{u} = 0$$

with $\mathbf{c} = \mathbf{u} - \mathbf{u}^G$, b.c. & i.c.

Space: Stabilized Finite Elements, **ALE**

$$\mathbf{M}^F \dot{\mathbf{u}} + \mathbf{N}^F(\mathbf{c})\, \mathbf{u} + \mathbf{G}^F \mathbf{p} = \mathbf{f}^F_{ext}$$

Time: One–step–Θ, Backward Euler, ...

...on moving mesh (pseudo–structure)

$$\mathbf{K}^M \mathbf{r} = \mathbf{f}^M(\mathbf{r}_\Gamma) \quad \text{with} \quad \mathbf{r}_\Gamma = \mathbf{d}_\Gamma$$

$$\mathbf{u}^G = \left(\mathbf{r}^{n+1} - \mathbf{r}^n\right)/\Delta t \qquad \text{((D)GCL!)}$$

Nonlinear Elastodynamics

Cauchy equation of motion:

$$\rho^S \ddot{\mathbf{d}} = \nabla \cdot (\mathbf{F} \cdot \mathbf{S}) + \rho^S \mathbf{b}$$

with b.c. & i.c.

Structural model:
Three–dimensional shell formulation

Space: Hybrid–mixed Finite Elements

$$\mathbf{M}^S \ddot{\mathbf{d}} \left(+ \mathbf{D}^S \dot{\mathbf{d}} \right) + \mathbf{N}^S(\mathbf{d}) = \mathbf{f}^S_{ext} - \mathbf{f}_\Gamma(\mathbf{p}_\Gamma, \boldsymbol{\tau}_\Gamma)$$

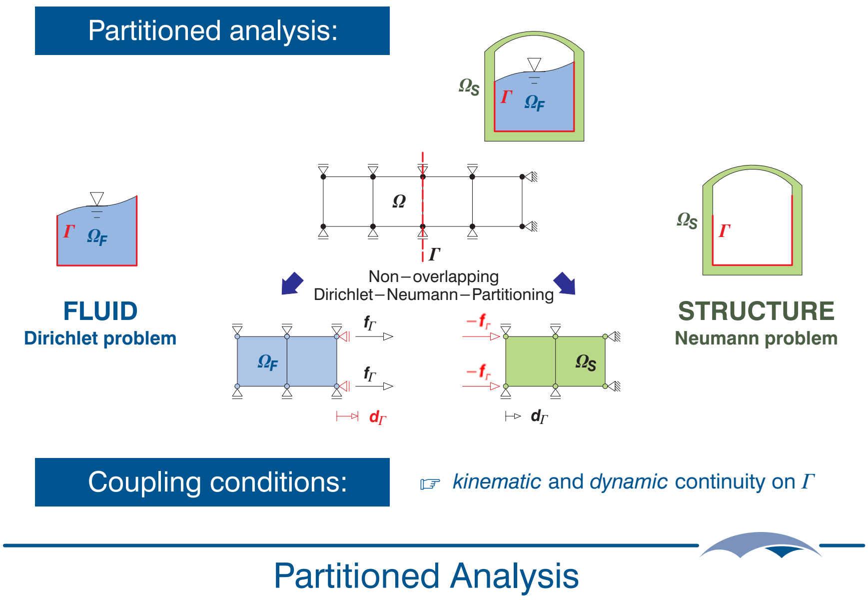
with $\boldsymbol{\tau}_\Gamma = \left(2\nu^F \boldsymbol{\epsilon}(\mathbf{u})\right)_\Gamma$

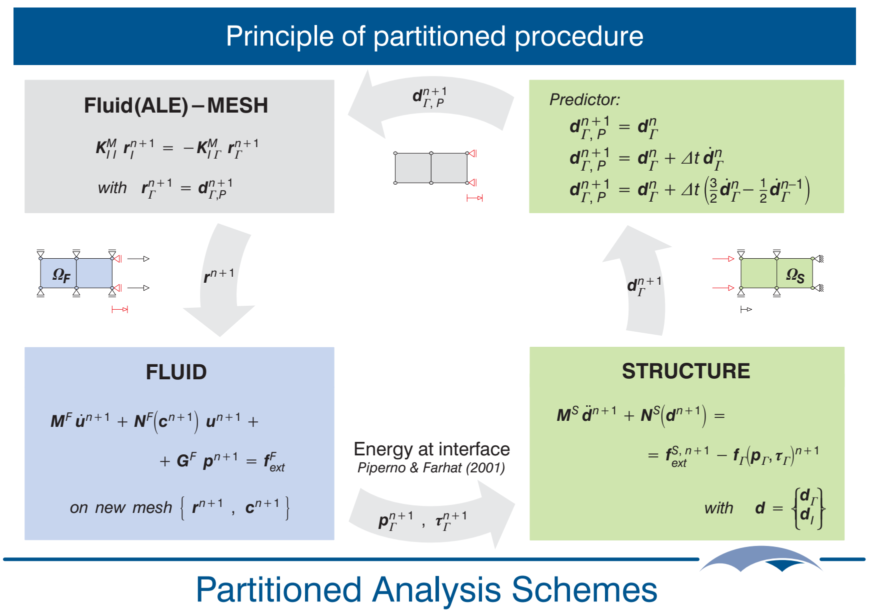
Time: Generalized–α, ...

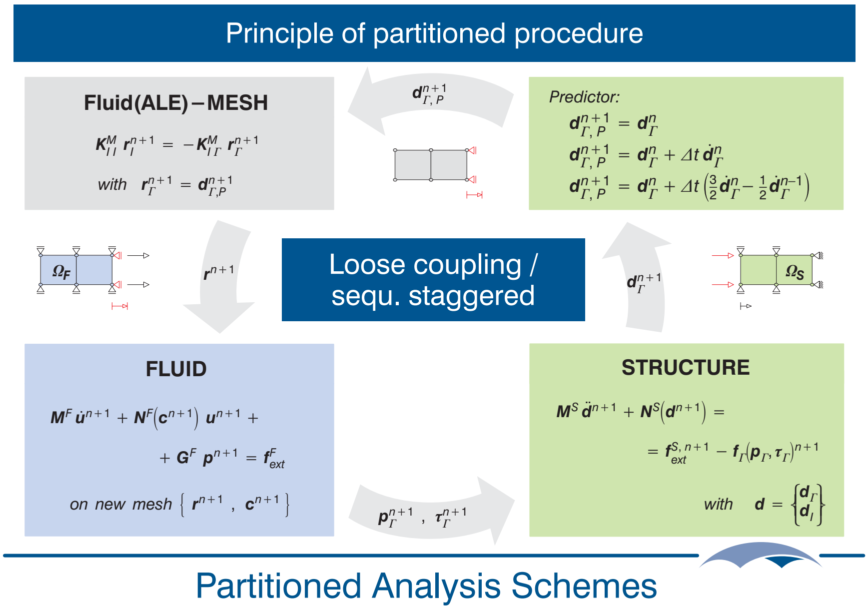
Single Field Solver

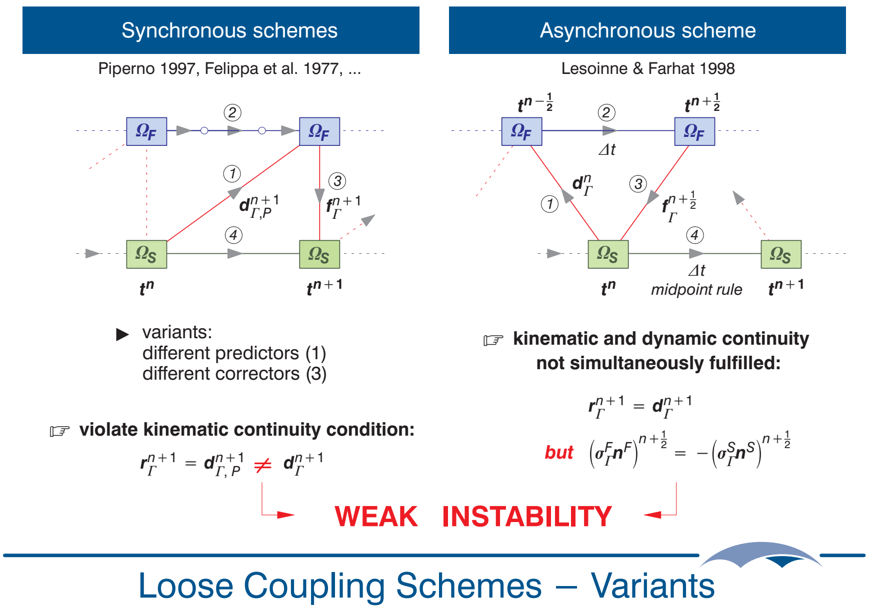
- Formulation
- **Coupling**
- Free Surface
- Turbulence
- Conclusions

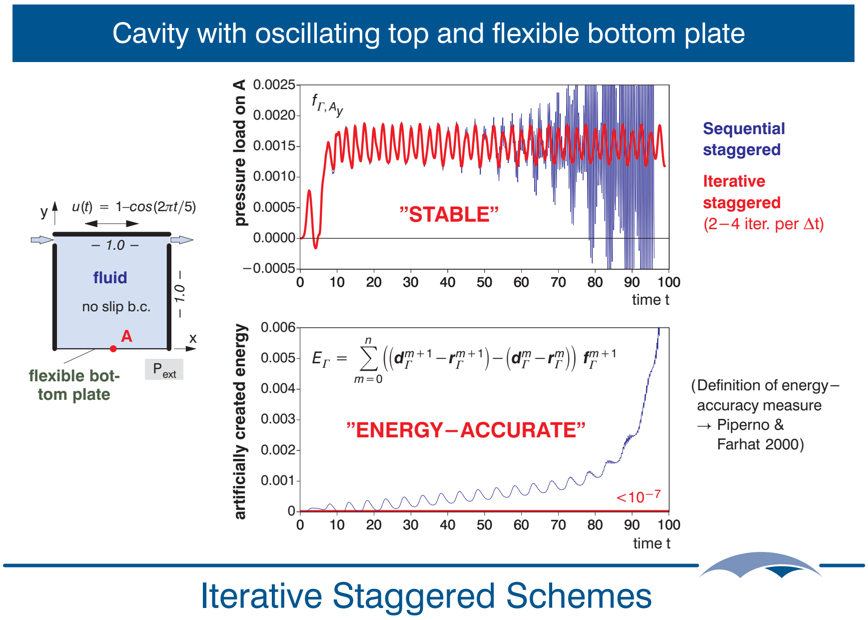
Outline

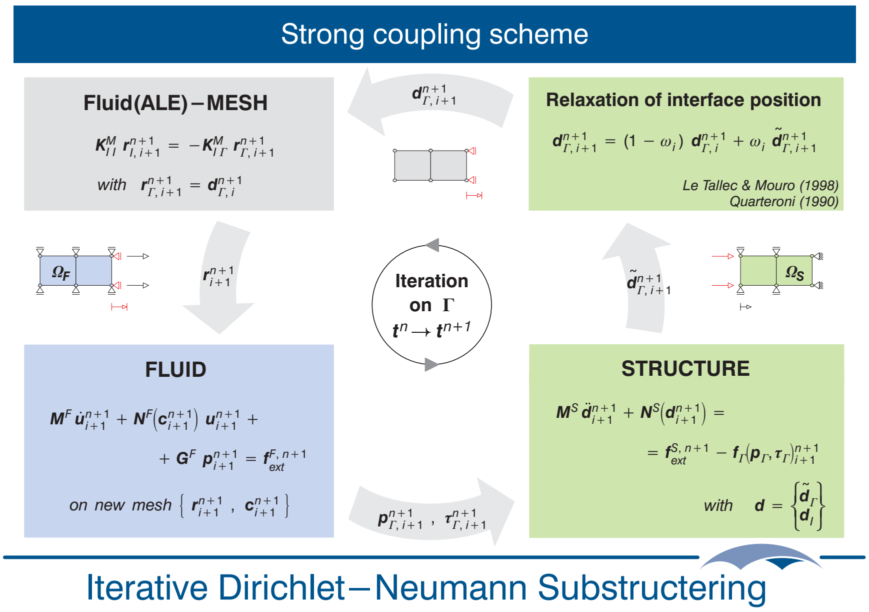

Partitioned analysis:
Ω_S
Γ
Ω_F
Ω
Γ
Non – overlapping
Dirichlet – Neumann – Partitioning
FLUID
Dirichlet problem
STRUCTURE
Neumann problem
f_Γ
$-f_\Gamma$
d_Γ
Coupling conditions:
kinematic and dynamic continuity on Γ
Partitioned Analysis

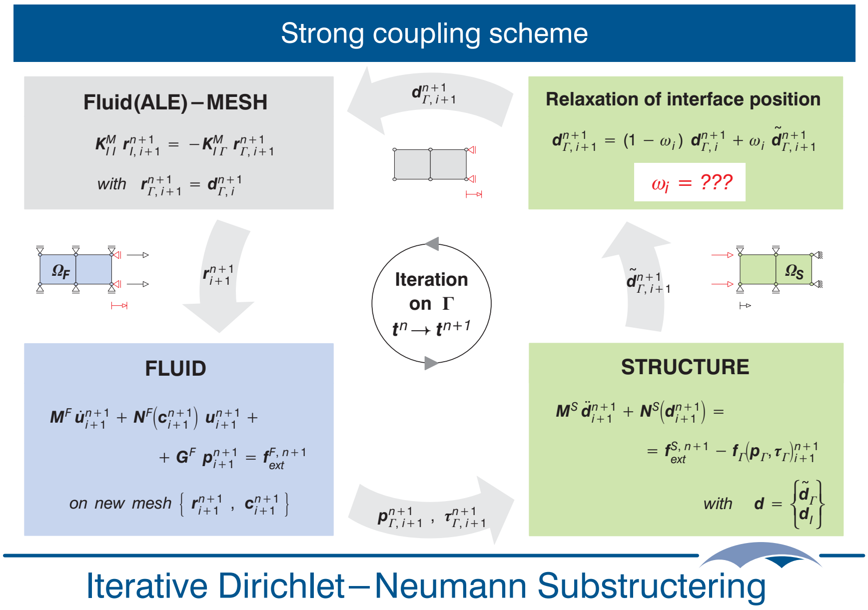

Principle of partitioned procedure
Fluid(ALE) – MESH
$K^M_{II}\, r^{n+1}_I = -K^M_{I\Gamma}\, r^{n+1}_\Gamma$
with $r^{n+1}_\Gamma = d^{n+1}_{\Gamma,P}$
$d^{n+1}_{\Gamma,P}$
Predictor:
$d^{n+1}_{\Gamma,P} = d^n_\Gamma$
$d^{n+1}_{\Gamma,P} = d^n_\Gamma + \Delta t\, \dot{d}^n_\Gamma$
$d^{n+1}_{\Gamma,P} = d^n_\Gamma + \Delta t \left(\frac{3}{2}\dot{d}^n_\Gamma - \frac{1}{2}\dot{d}^{n-1}_\Gamma\right)$
r^{n+1}
d^{n+1}_Γ
Ω_F
Ω_S
FLUID
$M^F \dot{u}^{n+1} + N^F(c^{n+1})\, u^{n+1} +$
$+ G^F p^{n+1} = f^F_{ext}$
on new mesh $\{ r^{n+1}, c^{n+1} \}$
Energy at interface
Piperno & Farhat (2001)
$p^{n+1}_\Gamma, \tau^{n+1}_\Gamma$
STRUCTURE
$M^S \ddot{d}^{n+1} + N^S(d^{n+1}) =$
$= f^{S,n+1}_{ext} - f_\Gamma(p_\Gamma, \tau_\Gamma)^{n+1}$
with $d = \begin{Bmatrix} d_\Gamma \\ d_I \end{Bmatrix}$
Partitioned Analysis Schemes

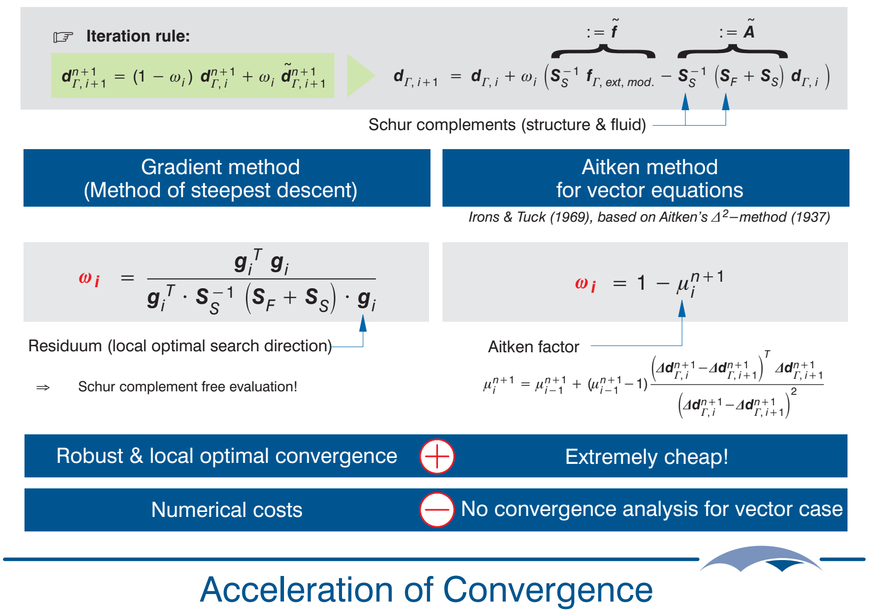

Principle of partitioned procedure
Fluid(ALE)–MESH
with
Predictor:
Loose coupling /
sequ. staggered
FLUID
on new mesh
STRUCTURE
with
Partitioned Analysis Schemes

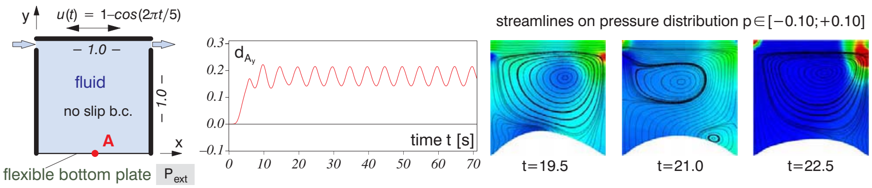

Synchronous schemes
Piperno 1997, Felippa et al. 1977, ...
Asynchronous scheme
Lesoinne & Farhat 1998
midpoint rule
variants:
different predictors (1)
different correctors (3)
violate kinematic continuity condition:
kinematic and dynamic continuity
not simultaneously fulfilled:
but
WEAK INSTABILITY
Loose Coupling Schemes – Variants


Cavity with oscillating top and flexible bottom plate
u(t) = 1−cos(2πt/5)
fluid
no slip b.c.
A
flexible bottom plate
P ext
pressure load on A
"STABLE"
time t
Sequential staggered
Iterative staggered
(2−4 iter. per Δt)
artificially created energy
"ENERGY−ACCURATE"
<10−7
(Definition of energy−accuracy measure → Piperno & Farhat 2000)
Iterative Staggered Schemes

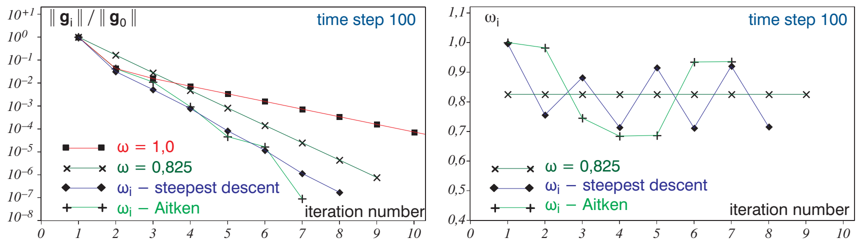

Strong coupling scheme
Fluid(ALE)−MESH
Relaxation of interface position
Le Tallec & Mouro (1998)
Quarteroni (1990)
Iteration on Γ
FLUID
on new mesh
STRUCTURE
with
Iterative Dirichlet−Neumann Substructering


Strong coupling scheme
Fluid(ALE) – MESH
$K^M_{II}\, r^{n+1}_{I,i+1} = -K^M_{I\Gamma}\, r^{n+1}_{\Gamma,i+1}$
with $r^{n+1}_{\Gamma,i+1} = d^{n+1}_{\Gamma,i}$
$d^{n+1}_{\Gamma,i+1}$
Relaxation of interface position
$d^{n+1}_{\Gamma,i+1} = (1-\omega_i)\, d^{n+1}_{\Gamma,i} + \omega_i\, \tilde{d}^{n+1}_{\Gamma,i+1}$
ω_i = ???
Ω_F
r^{n+1}_{i+1}
Iteration on Γ
$t^n \rightarrow t^{n+1}$
$\tilde{d}^{n+1}_{\Gamma,i+1}$
Ω_S
FLUID
$M^F \dot{u}^{n+1}_{i+1} + N^F(c^{n+1}_{i+1})\, u^{n+1}_{i+1} +$
$+ G^F p^{n+1}_{i+1} = f^{F,n+1}_{ext}$
on new mesh $\{ r^{n+1}_{i+1},\ c^{n+1}_{i+1} \}$
$p^{n+1}_{\Gamma,i+1},\ \tau^{n+1}_{\Gamma,i+1}$
STRUCTURE
$M^S \ddot{d}^{n+1}_{i+1} + N^S(d^{n+1}_{i+1}) =$
$= f^{S,n+1}_{ext} - f_\Gamma(p_\Gamma, \tau_\Gamma)^{n+1}_{i+1}$
with $d = \begin{Bmatrix} \tilde{d}_\Gamma \\ d_I \end{Bmatrix}$
Iterative Dirichlet – Neumann Substructering

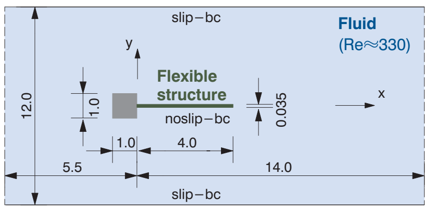

Iteration rule:
$d^{n+1}_{\Gamma,i+1} = (1-\omega_i)\, d^{n+1}_{\Gamma,i} + \omega_i\, \tilde{d}^{n+1}_{\Gamma,i+1}$
$d_{\Gamma,i+1} = d_{\Gamma,i} + \omega_i \left(\underbrace{S_S^{-1} f_{\Gamma,\text{ext, mod.}}}_{:=\tilde{f}} - \underbrace{S_S^{-1} (S_F + S_S)}_{:=\tilde{A}}\, d_{\Gamma,i} \right)$
Schur complements (structure & fluid)
Gradient method
(Method of steepest descent)
Aitken method
for vector equations
Irons & Tuck (1969), based on Aitken's Δ^2–method (1937)
$\omega_i = \dfrac{g_i^T\, g_i}{g_i^T \cdot S_S^{-1} (S_F + S_S) \cdot g_i}$
$\omega_i = 1 - \mu_i^{n+1}$
Residuum (local optimal search direction)
Aitken factor
⇒ Schur complement free evaluation!
$\mu_i^{n+1} = \mu_{i-1}^{n+1} + (\mu_{i-1}^{n+1} - 1) \dfrac{(\Delta d^{n+1}_{\Gamma,i} - \Delta d^{n+1}_{\Gamma,i+1})^T\, \Delta d^{n+1}_{\Gamma,i+1}}{(\Delta d^{n+1}_{\Gamma,i} - \Delta d^{n+1}_{\Gamma,i+1})^2}$
Robust & local optimal convergence
Extremely cheap!
Numerical costs
No convergence analysis for vector case
Acceleration of Convergence



ω – time step 1	0.00	0.25	0.50	0.70	0.85	0.90	1.00	1.20	≥1.42
Iterations	div.	45	19	11	**8**	11	15	34	div.

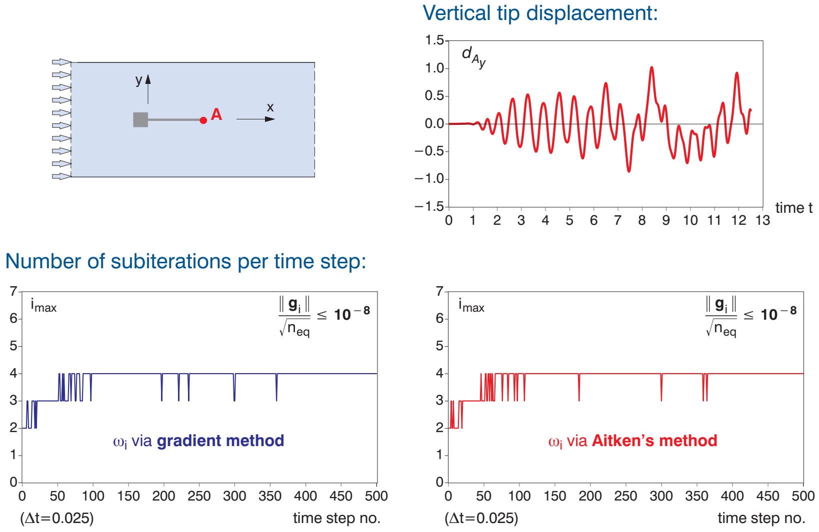


Convergence and Relaxation Parameter History

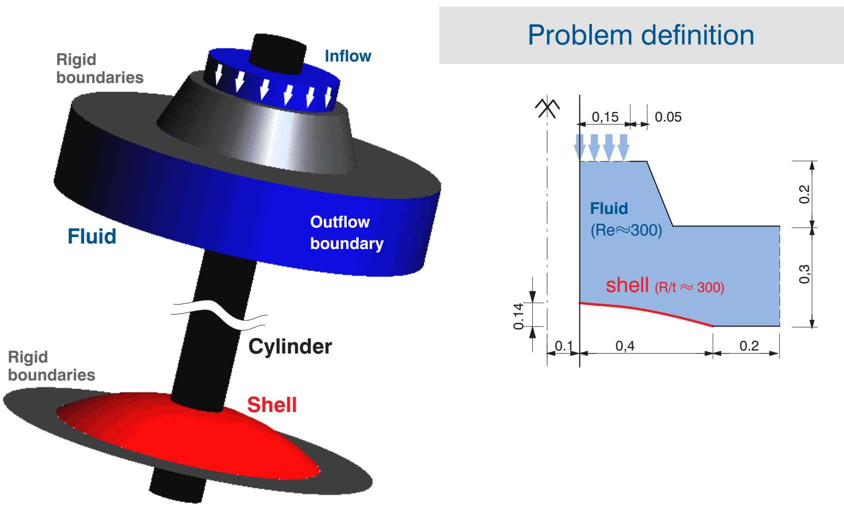


- Experiments for benchmarking

Vortex Induced Vibrations



Vortex Induced Vibrations



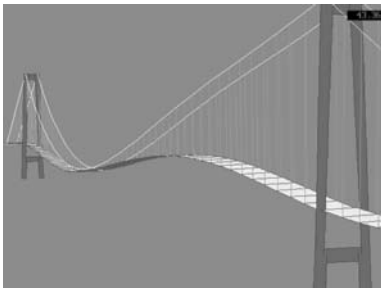
Snap through of Gasket

- Formulation
- Coupling
- **Free Surface**
- Turbulence
- Conclusions

Outline

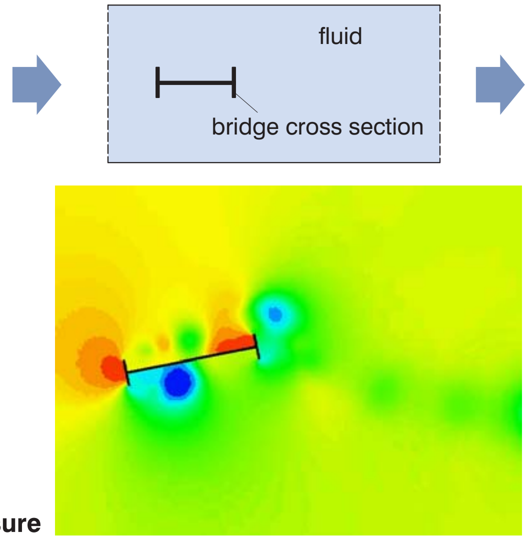
Tacoma Narrows Bridge, USA
(Collapse 7.11.1940)

Höga Kusten Bridge, Sweden
Simulation wind induced vibration



KOVACS, WEINSTADT 1995

Vibration of Bridges



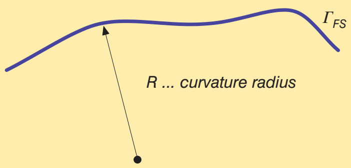
Tacoma Bridge Cross Section

Dynamic boundary conditions

Dynamic boundary condition

$$\sigma \cdot \boldsymbol{n} = \boldsymbol{h} \quad \text{on } \Gamma_{FS}$$

with $\sigma = -p\boldsymbol{I} + 2\nu\boldsymbol{\epsilon}(\boldsymbol{u})$



Balance of tractions at surface

$$\sigma \cdot \boldsymbol{n} = \sigma^a \cdot \boldsymbol{n} + \gamma\kappa\boldsymbol{n}$$

surface tension

stresses in air

Simplified definition:

$$\sigma \cdot \boldsymbol{n} = \gamma\kappa\boldsymbol{n} \quad \text{on } \Gamma_{FS}$$

γ ... surface tension coefficient

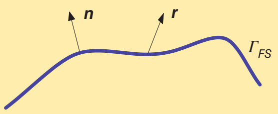
$\kappa = \frac{1}{R}$ ... curvature of free surface

Free Surface Flows

Kinematic boundary conditions

Kinematic boundary condition

$$\boldsymbol{u} \cdot \boldsymbol{n} = \boldsymbol{u}^G \cdot \boldsymbol{n} \quad \text{on } \Gamma_{FS}$$



n *... surface normal*

r *... grid displacement*

General elevation equation

$$-u_1 \frac{\partial r_3}{\partial x_1} - u_2 \frac{\partial r_3}{\partial x_2} + u_3 + u_1^G \frac{\partial r_3}{\partial x_1} + u_2^G \frac{\partial r_3}{\partial x_2} - u_3^G = 0$$

with $u_i^G = \frac{\partial r_i}{\partial t}$

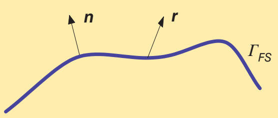
⇒ 'closure problem'

Free Surface Flows

Kinematic boundary conditions

Kinematic boundary condition

$$\boldsymbol{u} \cdot \boldsymbol{n} = \boldsymbol{u}^G \cdot \boldsymbol{n} \quad \text{on } \Gamma_{FS}$$



n *... surface normal*

r *... grid displacement*

General elevation equation

$$-u_1 \frac{\partial r_3}{\partial x_1} - u_2 \frac{\partial r_3}{\partial x_2} + u_3 + u_1^G \frac{\partial r_3}{\partial x_1} + u_2^G \frac{\partial r_3}{\partial x_2} - u_3^G = 0$$

with $u_i^G = \frac{\partial r_i}{\partial t}$

⇒ 'closure problem'

local Lagrangean approach

$$\boldsymbol{u} - \boldsymbol{u}^G = 0$$

height function approach ⇒ $r_1 = r_2 \equiv 0$

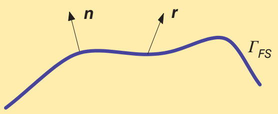
$$\frac{\partial r_3}{\partial t} + u_1 \frac{\partial r_3}{\partial x_1} + u_2 \frac{\partial r_3}{\partial x_2} - u_3 = 0$$

Free Surface Flows

Kinematic boundary conditions

Kinematic boundary condition

$$\boldsymbol{u} \cdot \boldsymbol{n} = \boldsymbol{u}^G \cdot \boldsymbol{n} \quad \text{on } \Gamma_{FS}$$



n ... *surface normal*

r ... *grid displacement*

General elevation equation

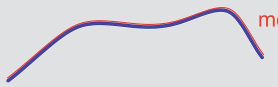
$$-u_1 \frac{\partial r_3}{\partial x_1} - u_2 \frac{\partial r_3}{\partial x_2} + u_3 + u_1^G \frac{\partial r_3}{\partial x_1} + u_2^G \frac{\partial r_3}{\partial x_2} - u_3^G = 0$$

$$\text{with} \quad u_i^G = \frac{\partial r_i}{\partial t}$$

⇒ 'closure problem'

For general cases (curved boundaries, fsi, ...)

⇒ general elevation equation + dimensionally reduced pseudo−structural approach



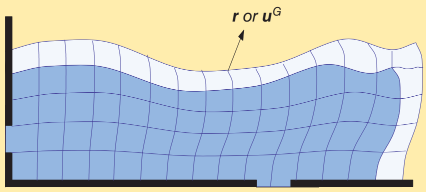
Free Surface Flows

Partitioned implicit

▶ Idea: Decomposition of fluid domain

$$\Omega = \Omega_{int} \cup \Omega_{FS}$$

element layer or boundary layer with constraints



▶ *Introduce* $\boldsymbol{r}$ *or* $\boldsymbol{u}^G$ *as additional unknown on* Γ_{FS} *and include weak form of the respective kinematic boundary condition on* Γ_{FS}

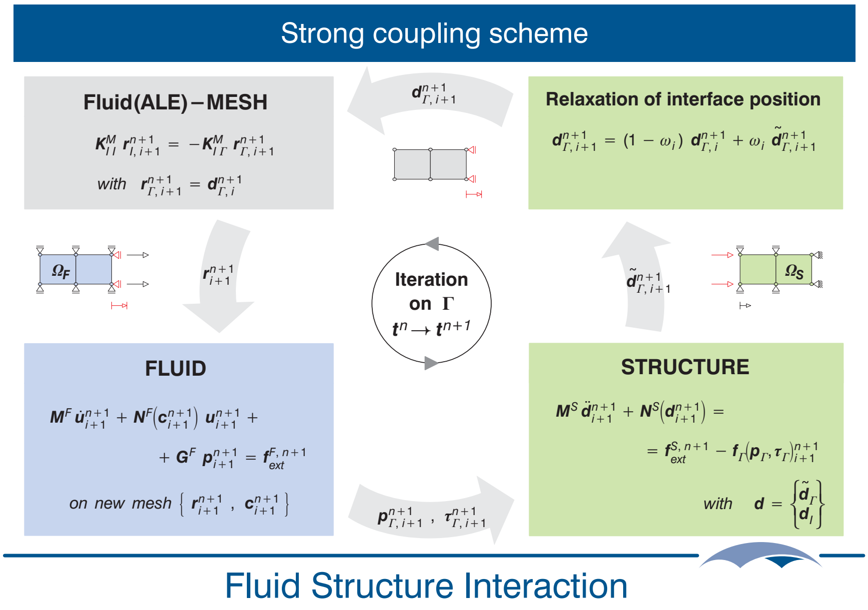
plus stabilization as additional term in variational formulation

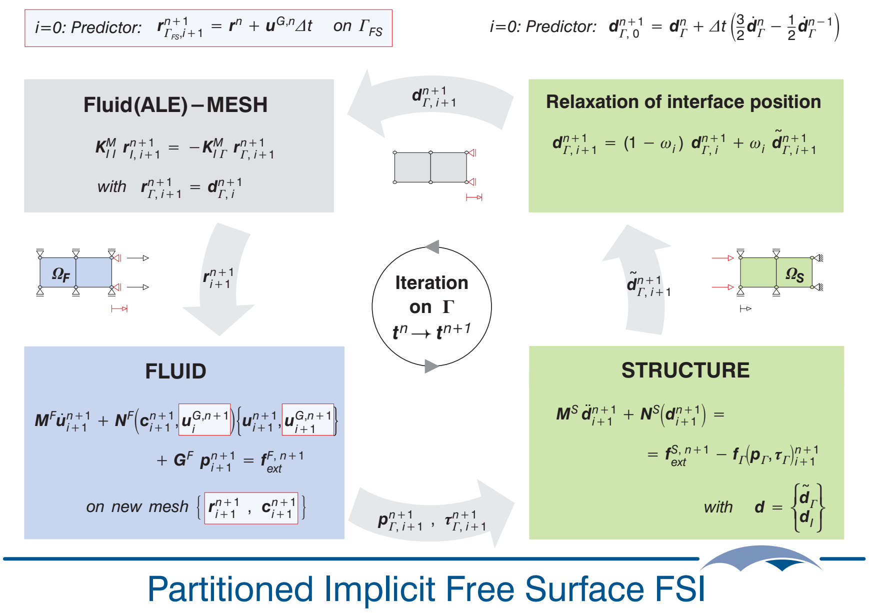
$$-u_1 \frac{\partial r_3}{\partial x_1} - u_2 \frac{\partial r_3}{\partial x_2} + u_3 + u_1^G \frac{\partial r_3}{\partial x_1} + u_2^G \frac{\partial r_3}{\partial x_2} - u_3^G = 0$$

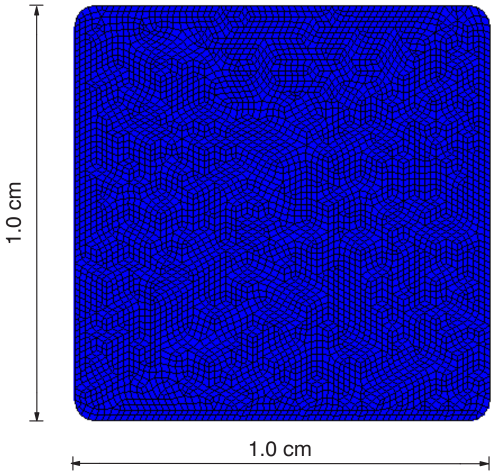
$$u_i - u_i^G = 0$$

$$\frac{\partial r_3}{\partial t} + u_1 \frac{\partial r_3}{\partial x_1} + u_2 \frac{\partial r_3}{\partial x_2} - u_3 = 0$$

Partitioned Implicit Free Surface Algorithm


Strong coupling scheme
Fluid(ALE)–MESH
Relaxation of interface position
Iteration on Γ
FLUID
STRUCTURE
Fluid Structure Interaction


i=0: Predictor:
Fluid(ALE)–MESH
Relaxation of interface position
Iteration on Γ
FLUID
STRUCTURE
Partitioned Implicit Free Surface FSI



Material Parameter:

$\varrho = 1.0 \frac{g}{cm^3}$

$\nu = 1.0 \frac{g}{cms}$

$\gamma = 73.0 \frac{g}{s^2}$

Boundary Conditions:

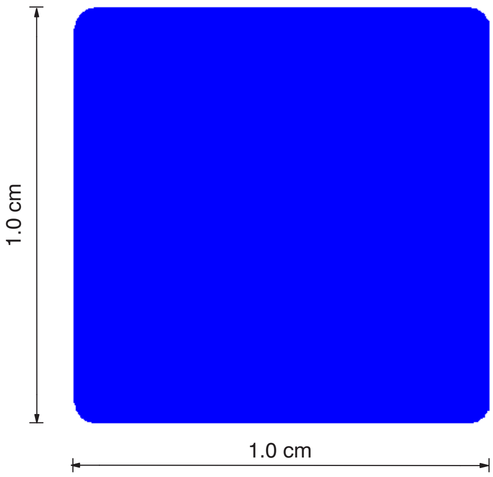
– *no Dirichlet–BCs*

– *include surface tension*

Discretisation:

10363 Q1Q1 fluid elements

Example: Nonequilibrium Rod



Material Parameter:

$\varrho = 1.0 \frac{g}{cm^3}$

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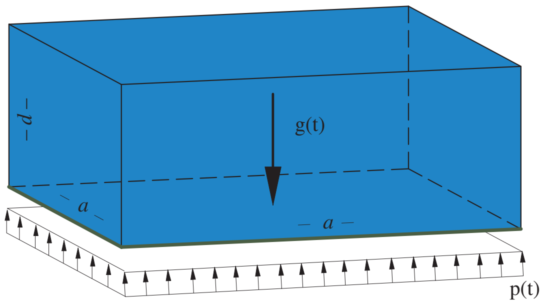
Example: Nonequilibrium Rod



Geometry:

$a = 10.0\ cm$

$d = 10.0\ cm$

Discretisation:

20 x 20 x 8 Q1Q1 fluid elements

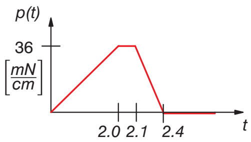
20 x 20 linear elastic shell elements

Material Parameter (Oil):

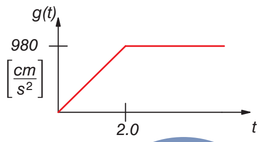
$\varrho = 0.92 \frac{g}{cm^3}$

$\nu = 9.0 \frac{cm^2}{s}$

External Load:



Gravity:



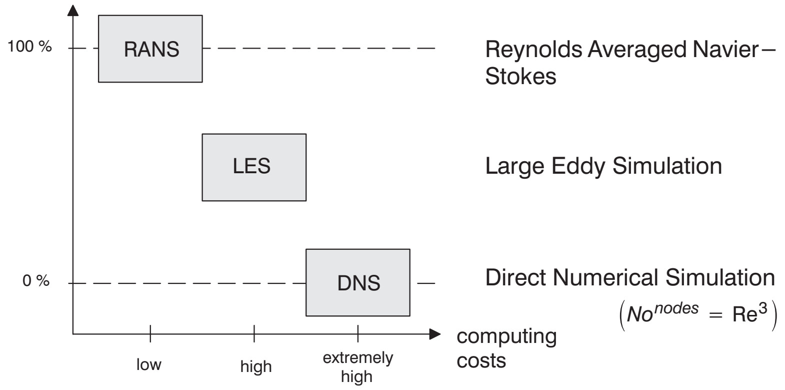
Example: Tank with flexible bottom plate

- Formulation
- Coupling
- Free Surface
- **Turbulence**
- Conclusions

Outline

Schematic Classification of Alternative Procedures

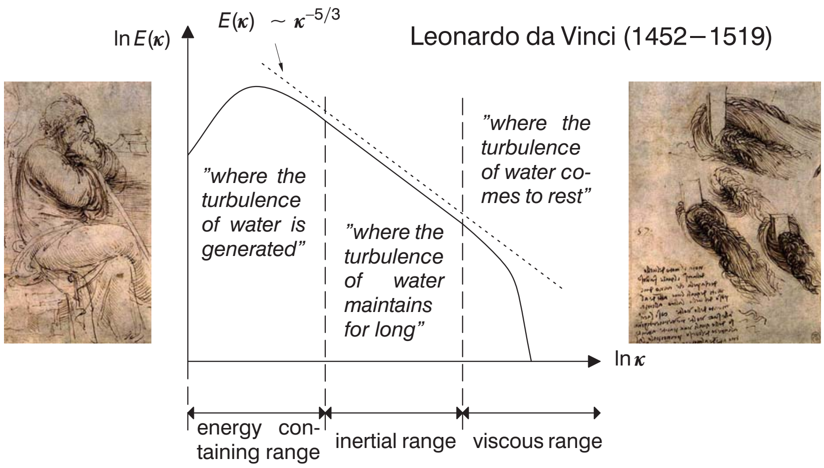
(Breuer 2000)



Simulation of Turbulent Flows

Kolmogorov Energy Spectrum

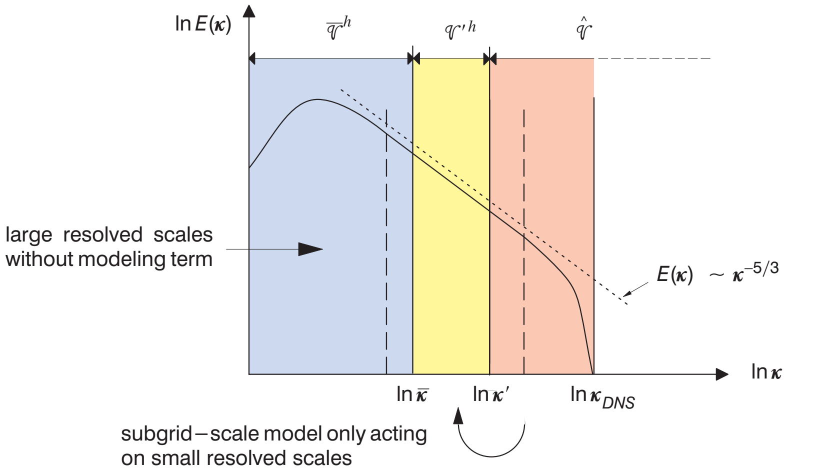
(Kolmogorov 1941)



Energy Spectrum of Turbulent Flows

Variational Multiscale Method (VMM)

(Hughes et al. 1998–2004)



VMM – Separation of 3 Scales

Variational multiscale method for Navier–Stokes equations (3 scales)

(Collis 2001, Gravemeier 2003)

separ. of function space $\mathcal{V}_u = \overline{\mathcal{V}}_u^h \oplus \mathcal{V}'_u \oplus \hat{\mathcal{V}}_u$ (large (resolved) scales; small (resolved) scales; unresolved scales)

separ. of solution and weighting functions

$$u = \overline{u} + u' + \hat{u} \qquad w = \overline{w} + w' + \hat{w}$$
$$p = \overline{p} + p' + \hat{p} \qquad q = \overline{q} + q' + \hat{q}$$

3 subproblems

$$B_{NS}\left(\overline{v}^h, \overline{q}^h; \overline{u}^h + u' + \hat{u}, \overline{p}^h + p' + \hat{p}\right) = \left(\overline{v}^h, f\right)_\Omega + \left(\overline{v}^h, h\right)_{\Gamma_h}$$
→ large–scale eq.

$$B_{NS}\left(v', q'; \overline{u}^h + u' + \hat{u}, \overline{p}^h + p' + \hat{p}\right) = (v', f)_\Omega + (v', h)_{\Gamma_h}$$
→ small–scale eq.

$$B_{NS}\left(\hat{v}, \hat{q}; \overline{u}^h + u' + \hat{u}, \overline{p}^h + p' + \hat{p}\right) = \left(\hat{v}, f\right)_\Omega + \left(\hat{v}, h\right)_{\Gamma_h}$$
→ equation for the unresolved scales of the problem

no solution, modeling of dissipative effect on small scales

VMM – Separation of 3 Scales

Variational multiscale method for Navier−Stokes equations (3 scales)

(Collis 2001, Gravemeier 2003)

large (resolved) scales
small (resolved) scales
unresolved scales

separ. of function space $\mathcal{V}_u = \bar{\mathcal{V}}_u^h \oplus \mathcal{V}'_u \oplus \hat{\mathcal{V}}_u$

separ. of solution and weighting functions $u = \bar{u} + u' + \hat{u}$ $\quad w = \bar{w} + w' + \hat{w}$

$p = \bar{p} + p' + \hat{p}$ $\quad q = \bar{q} + q' + \hat{q}$

3 subproblems

$B_{NS}\left(\bar{v}^h, \bar{q}^h; \bar{u}^h + u' \quad , \bar{p}^h + p' \quad \right) = \left(\bar{v}^h, f\right)_\Omega + \left(\bar{v}^h, h\right)_{\Gamma_h}$ ➡ modeled large−scale eq.

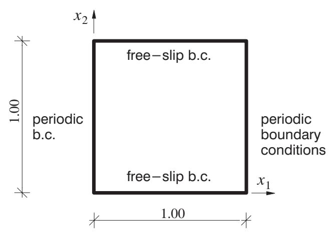
$B_{NS}\left(v', q'; \bar{u}^h + u', \bar{p}^h + p'\right) + D(\hat{u}, \hat{p}) = (v', f)_\Omega + (v', h)_{\Gamma_h}$ ➡ modeled small−scale eq.

equation for the unresolved scales of the problem

no solution, modeling of dissipative effect on small scales

VMM – Separation of 3 Scales

Plane mixing layer

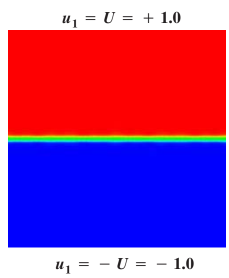


initial velocity distribution:

hyperbolic tangent profile

$u_1(x_2) = U \tanh\left(\frac{2x_2 - 1}{\delta_0}\right)$

initial vorticity thickness δ_0



$\nu = 3.57 \cdot 10^{-6}$ $\quad \delta_0 = \frac{1}{28}$ $\quad U = 1.0$

$\Rightarrow \mathbf{Re} = \mathbf{10,000}$

spatial discretization: 40x40 – 240x240 Q1Q1 elements

temporal discretization: $\delta t = 0.0125$ (570 time steps)

Turbulent Flow Example

- Formulation
- Coupling
- Free Surface
- Turbulence
- **Conclusions**

Outline

- **Multifield > Σ (Single Fields $_i$)**
- **Pure FE Approach**
- **Iterative Strong Coupling**
- **Implicit Free Surface**
- **Variational Multiscale Method for Turbulent Flows (LES)**

Conclusion