

Nonlinear viscoelastic modeling of foams

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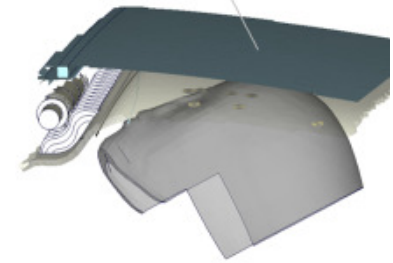


Outline

- Motivation
- State of the art in LS-DYNA®
- Nonlinear viscoelasticity for foams
- Modeling of Confor® foam
- Summary
- Literature

Motivation

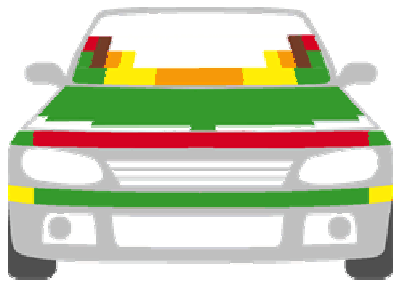
- Polymer materials and passive safety



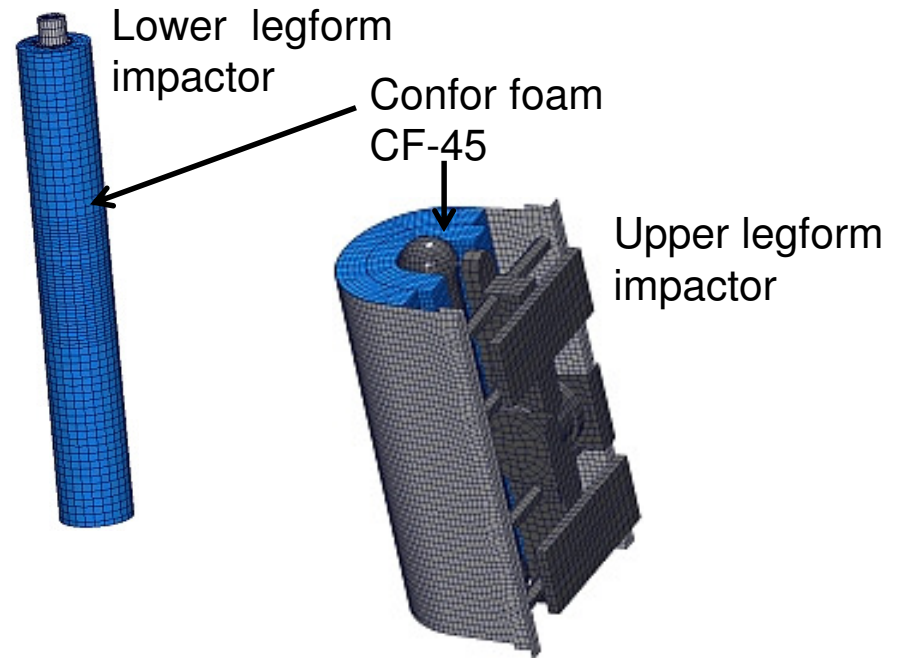
[www.daimler.com]

- Pedestrian protection

Euro NCAP
safety rating



[www.euroncap.org]





Outline

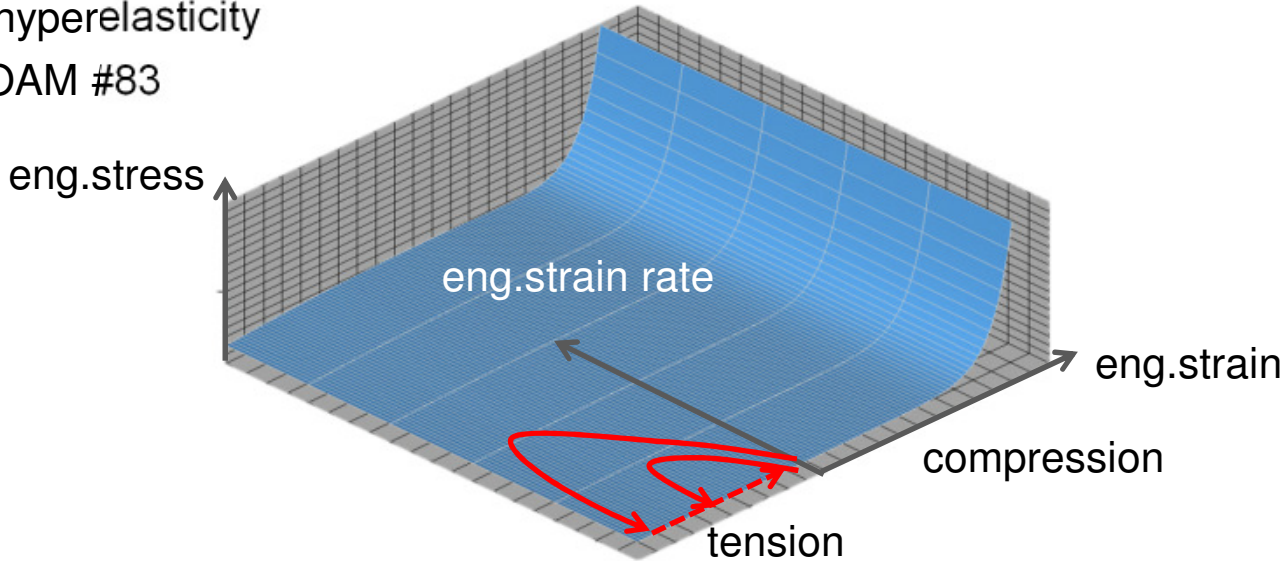
- Motivation
- **State of the art in LS-DYNA®**
- Nonlinear viscoelasticity for foams
- Modeling of Confor® foam
- Summary



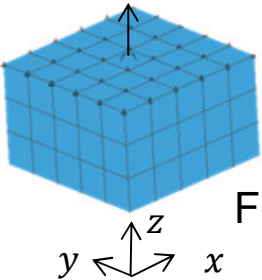
State of the art in LS-DYNA®

- Strain rate dependent hyperelasticity
*MAT_FU_CHANG_FOAM #83

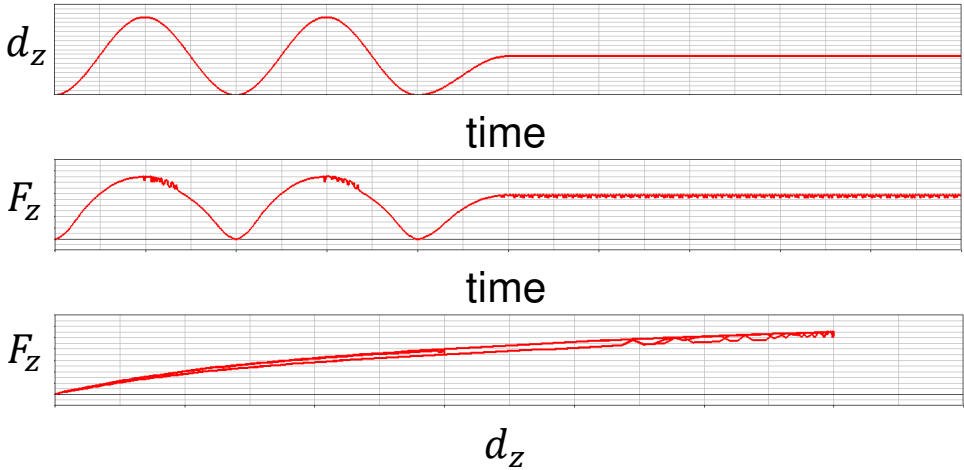
Without damage



Time-dependent displacement d_z of the upper surface



Fixed lower surface



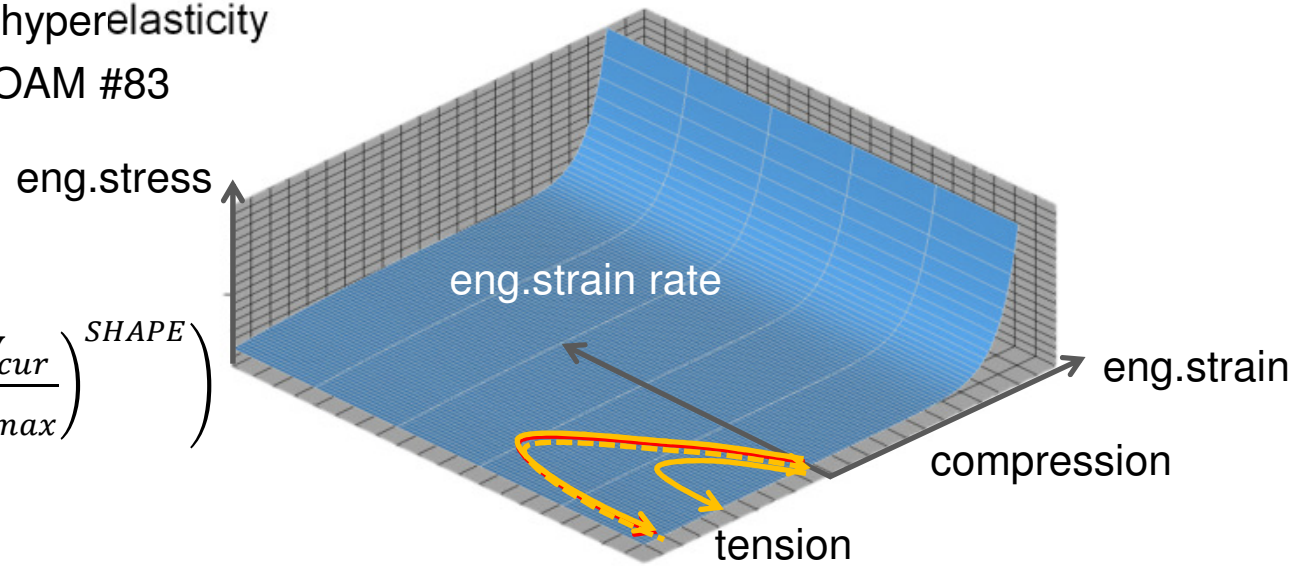
State of the art in LS-DYNA®

- Strain rate dependent hyperelasticity
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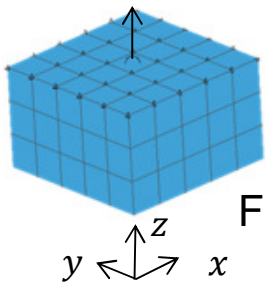
With damage

$$\sigma_i = \sigma_i(1 - d)$$

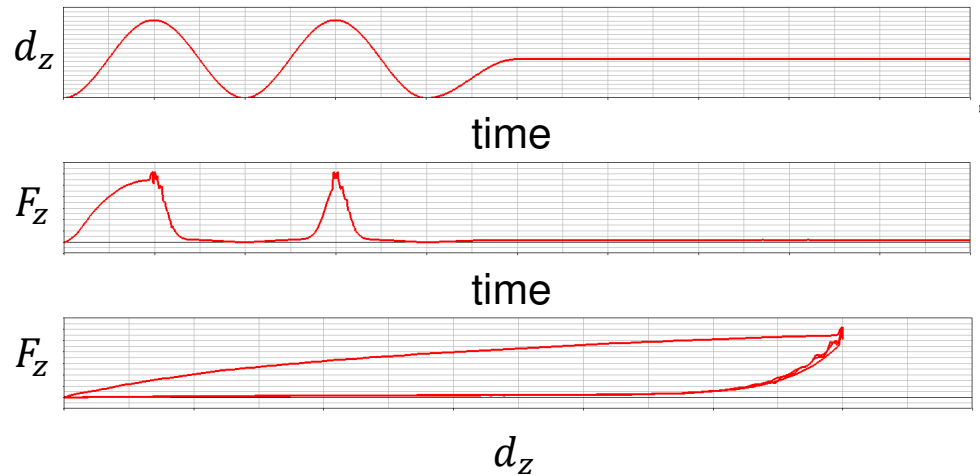
$$d = (1 - HU) \left(1 - \left(\frac{W_{cur}}{W_{max}} \right)^{SHAPE} \right)$$



Time-dependent displacement d_z of the upper surface



Fixed lower surface





State of the art in LS-DYNA®

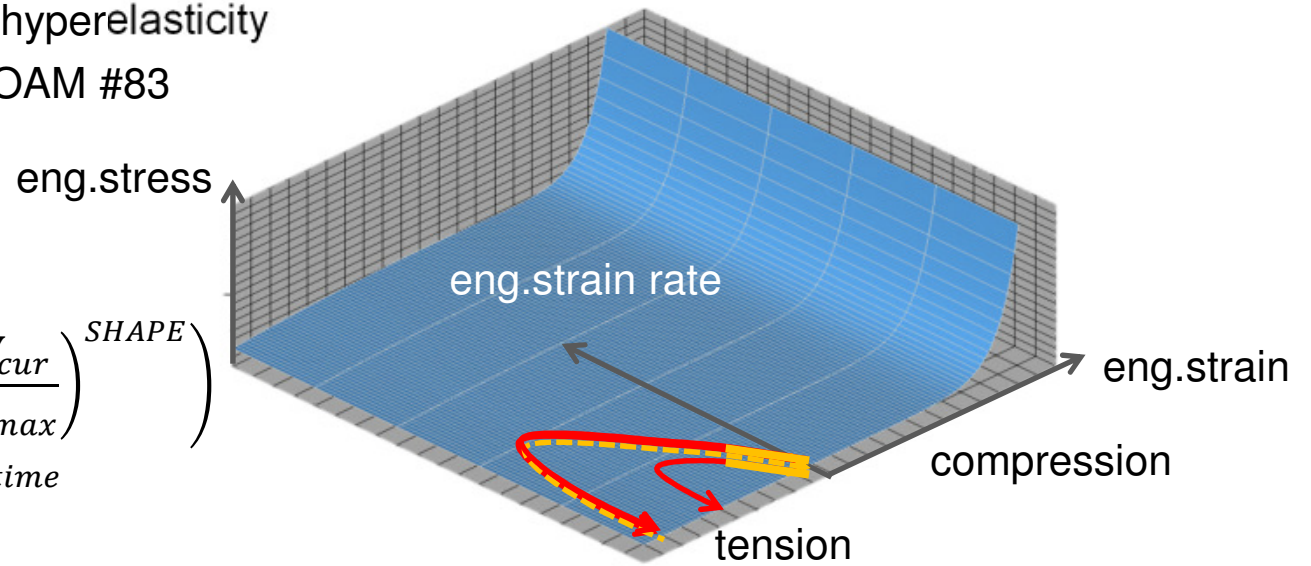
- Strain rate dependent hyperelasticity
*MAT_FU_CHANG_FOAM #83

With damage decay

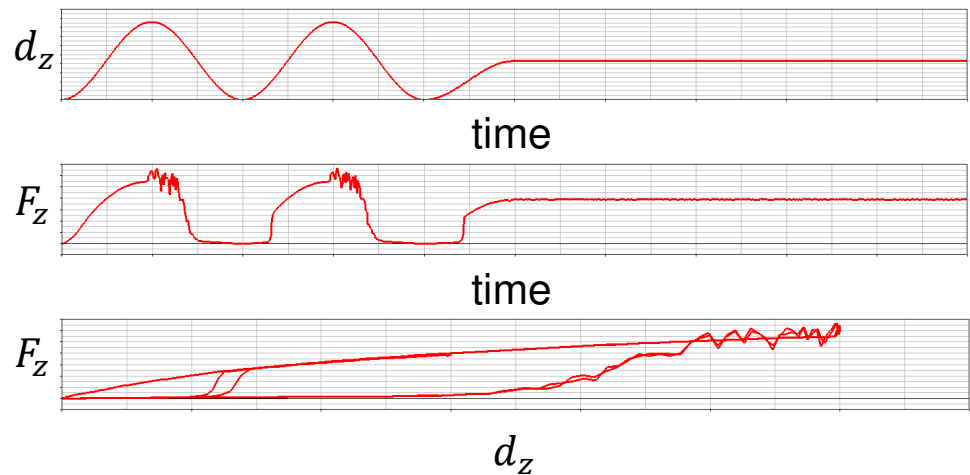
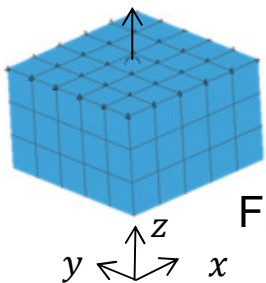
$$\sigma_i = \sigma_i(1 - d) \left(1 - \left(\frac{W_{cur}}{W_{max}} \right)^{SHAPE} \right)$$

$$d = (1 - HU) \left(1 - \left(\frac{W_{cur}}{W_{max}} \right)^{SHAPE} \right)$$

$$W_{max} = W_{max} e^{-BETAT \cdot time}$$



Time-dependent displacement d_z of the upper surface

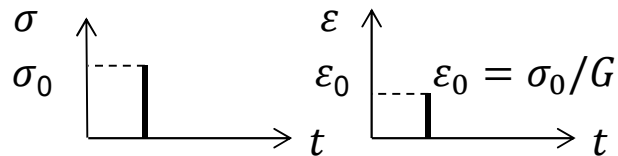




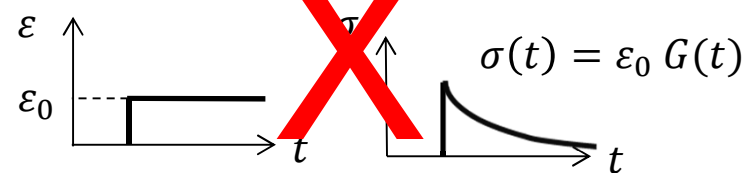
State of the art in LS-DYNA®

- Characteristic properties of viscoelastic solids

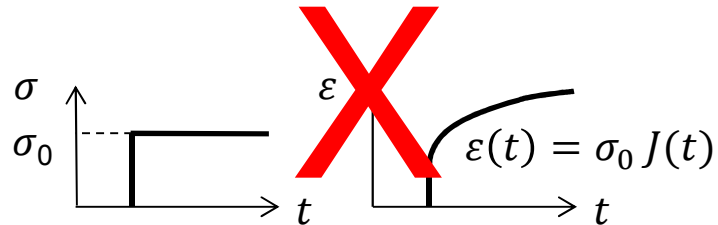
Instantaneous elasticity



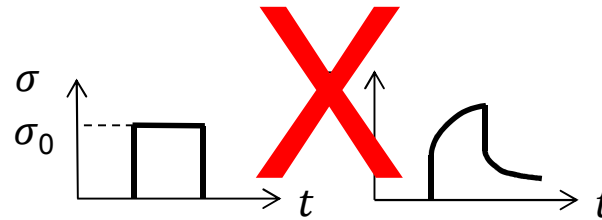
Stress relaxation under constant strain



Creep under constant stress



Instantaneous and delayed recovery



Strain rate dependent hyperelasticity
with damage formulation *MAT_083

State of the art in LS-DYNA®

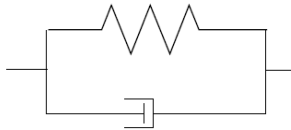
- Linear viscoelastic material models based on rheological models

Material models: 6, 61, 76, 86, 134, 164, 234, 276,...

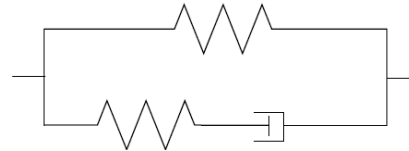
Maxwell-Element



Kelvin-Voigt-Element



Standard Linear Solid



Generalized Maxwell element of linear viscoelasticity

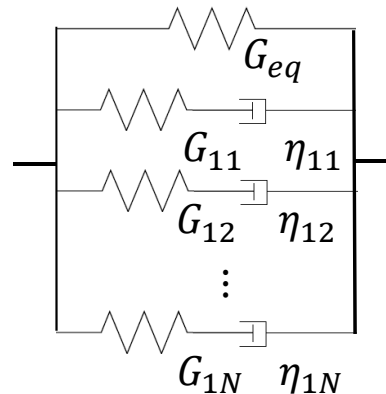
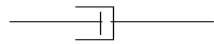
Hooke's law

$$\sigma(t) = G\varepsilon(t)$$



Newton's law

$$\sigma(t) = \eta \frac{\partial \varepsilon(t)}{\partial t}$$



Prony series

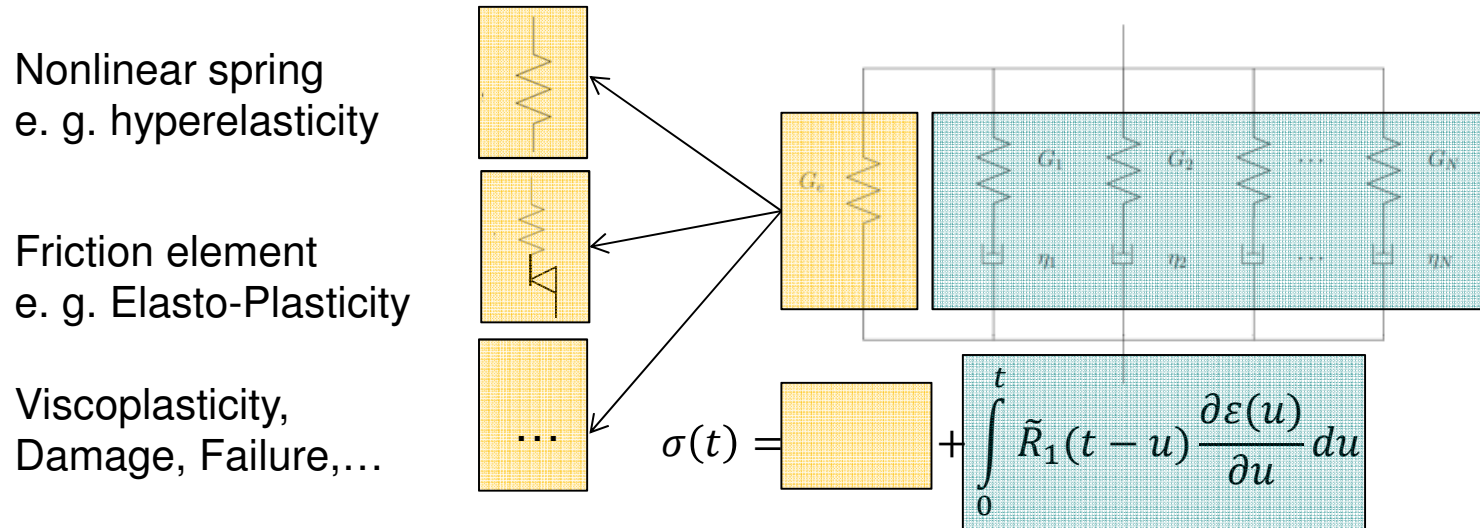
$$\tilde{R}_1(t) = G_{eq} + \sum_{i=1}^N G_{1i} e^{-\beta_{1i} t}$$

$$\beta_{1i} = \eta_{1i} / G_{1i}$$

$$\sigma(t) = \int_0^t \tilde{R}_1(t-u) \frac{\partial \varepsilon(u)}{\partial u} du$$

State of the art in LS-DYNA®

- Linear viscoelastic overstress and equilibrium material model



Material models: Hyperelasticity, (Visco-) Plasticity, ...
57, 73, 77, 87, 91, 124, 127, 129, 155, 158, 175, 178,...



State of the art in LS-DYNA®

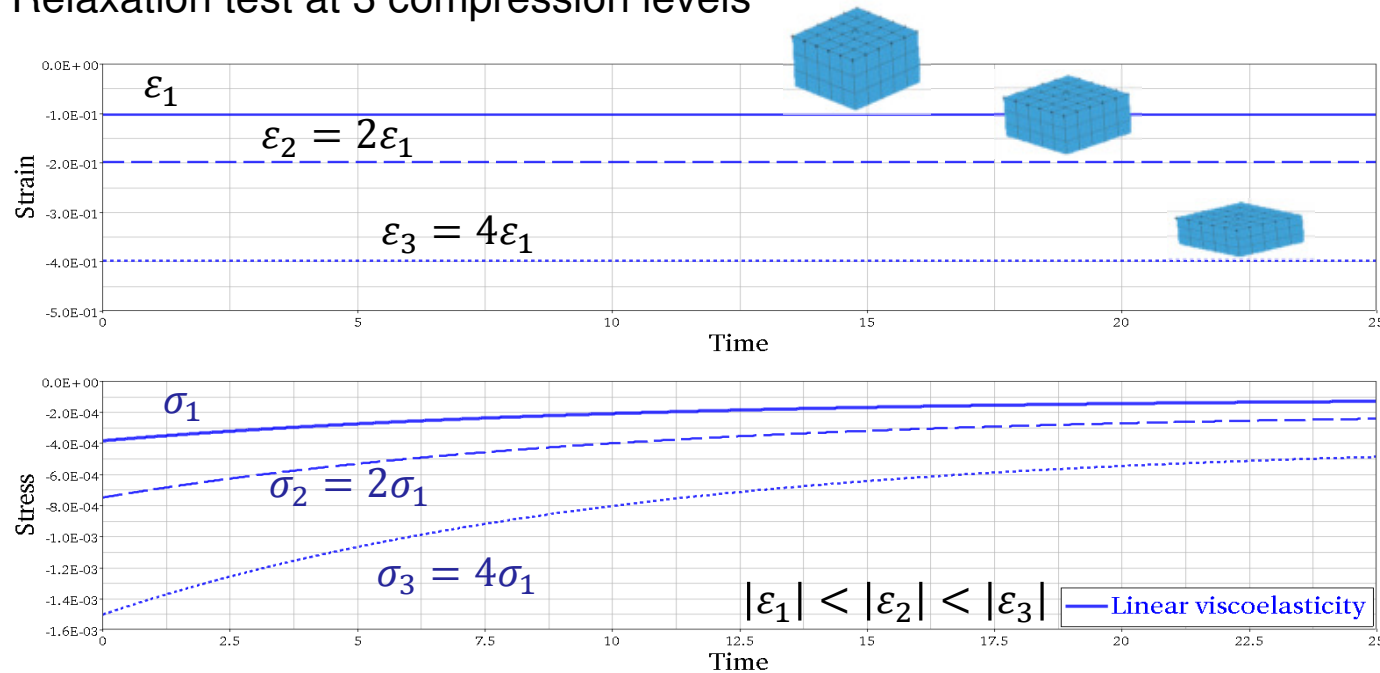
- Linear viscoelasticity for simple, homogeneous and non-aging materials

$$\sigma(t) = \mathcal{F}_{u=0}^t(\varepsilon(t - u))$$

- Stress-Strain Linearity

$$\begin{aligned} \mathcal{F}_{u=0}^t(\alpha\varepsilon(t - u)) &= \alpha\mathcal{F}_{u=0}^t(\varepsilon(t - u)) \\ &= \alpha\sigma(t) \end{aligned}$$

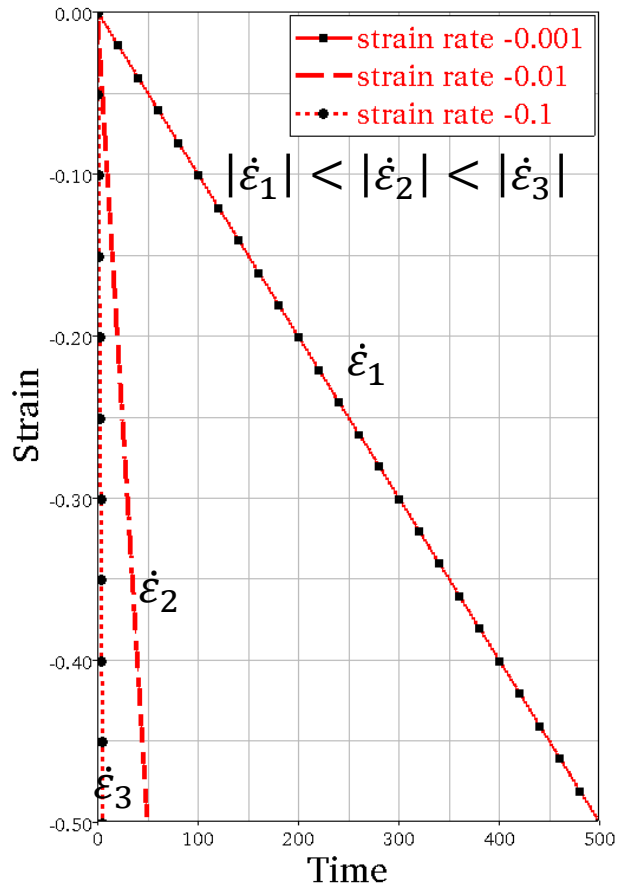
- Relaxation test at 3 compression levels





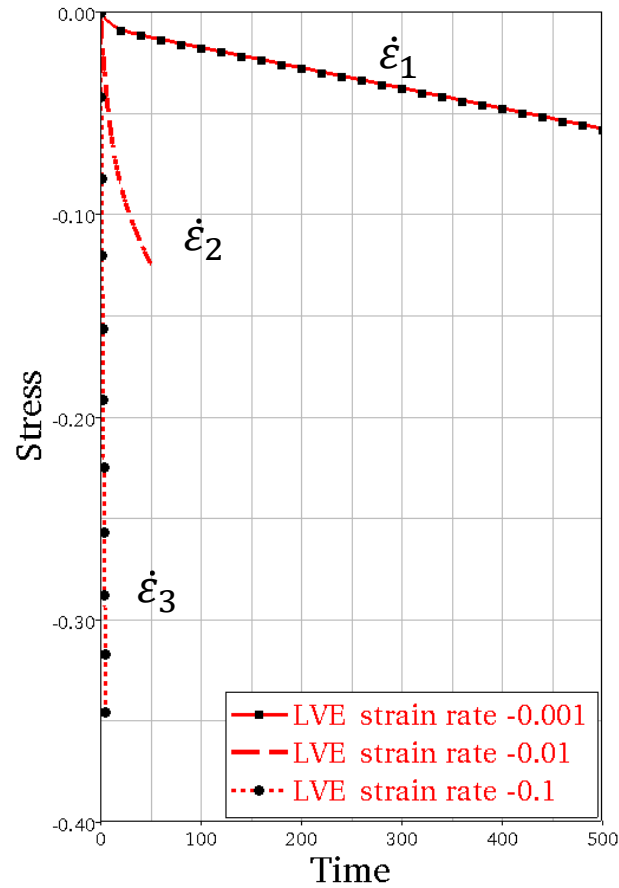
State of the art in LS-DYNA®

- Linear viscoelasticity: constant strain rate excitation



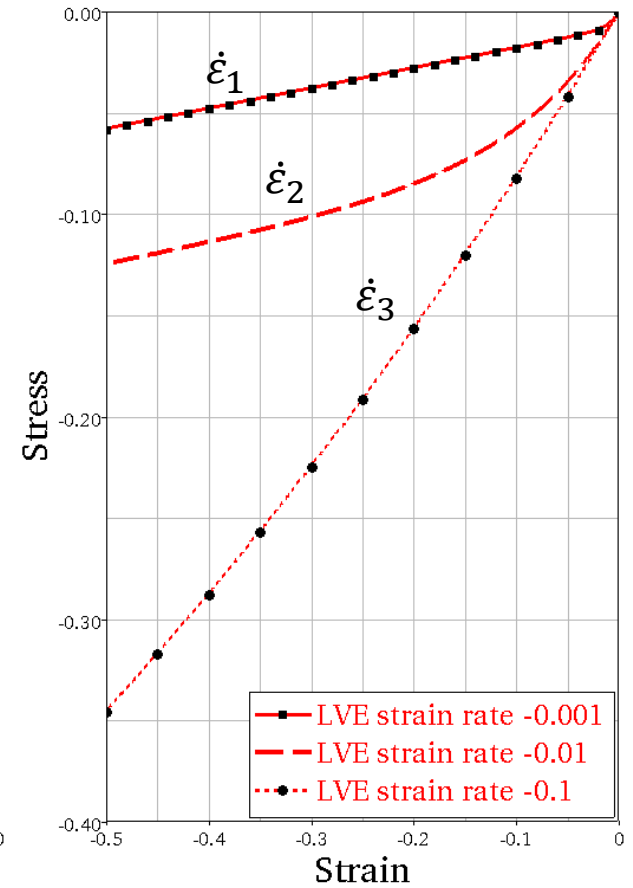
$$\epsilon_1(t) = \dot{\epsilon}_1 t$$

$$\epsilon_2(t) = \alpha \dot{\epsilon}_1 t$$



$$\sigma_1(t)$$

$$\sigma_2(t) = \alpha \sigma_1(t)$$





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- Modeling of Confor® foam
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Nonlinear viscoelasticity for foams

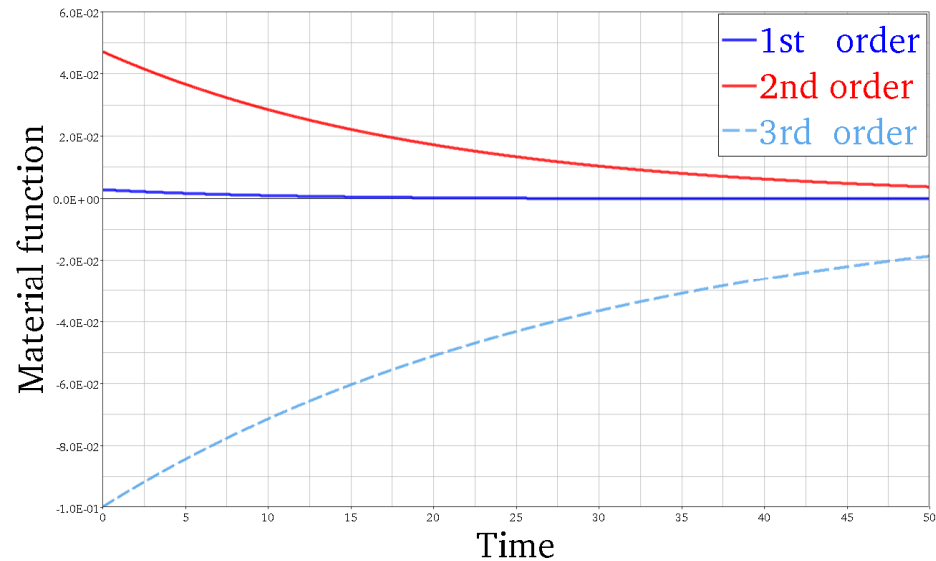
- Green-Rivlin model for isotropic foams

$$\begin{aligned}
 \mathbf{S}(t) = & \int_0^t \tilde{R}_1(t - u_1) \dot{\mathbf{E}}(u_1) du_1 + \left[\int_0^t \tilde{R}_2(t - u_1) \dot{\mathbf{E}}(u_1) du_1 \right]^2 \\
 & + \left[\int_0^t \tilde{R}_3(t - u_1) \dot{\mathbf{E}}(u_1) du_1 \right]^3 + \left[\int_0^t \tilde{R}_4(t - u_1) \dot{\mathbf{E}}(u_1) du_1 \right]^4 \\
 & + \left[\int_0^t \tilde{R}_5(t - u_1) \dot{\mathbf{E}}(u_1) du_1 \right]^5
 \end{aligned}$$

Prony series up to 5th order

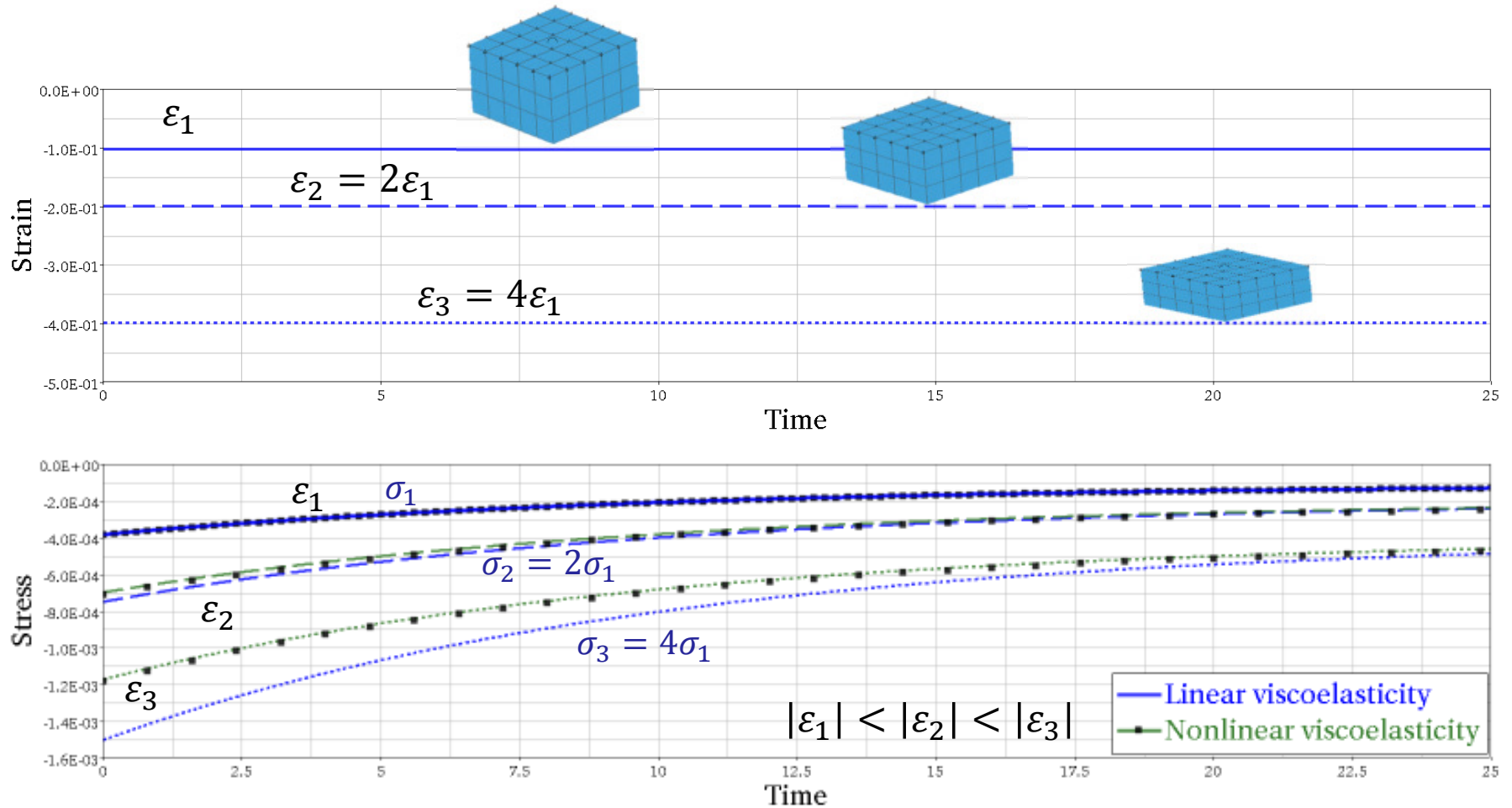
$$\tilde{R}_j(t) = \sum_{i=1}^N G_{ji} e^{-\beta_{ji} t}$$

$$[\tilde{R}_j(t)] = \sqrt[j]{N/m^2}$$



Nonlinear viscoelasticity for foams

- Relaxation test at 3 compression level: Saturation



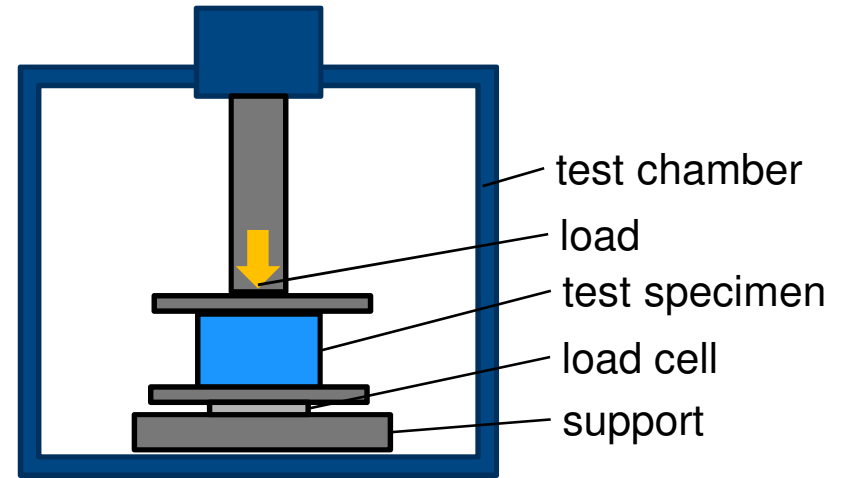
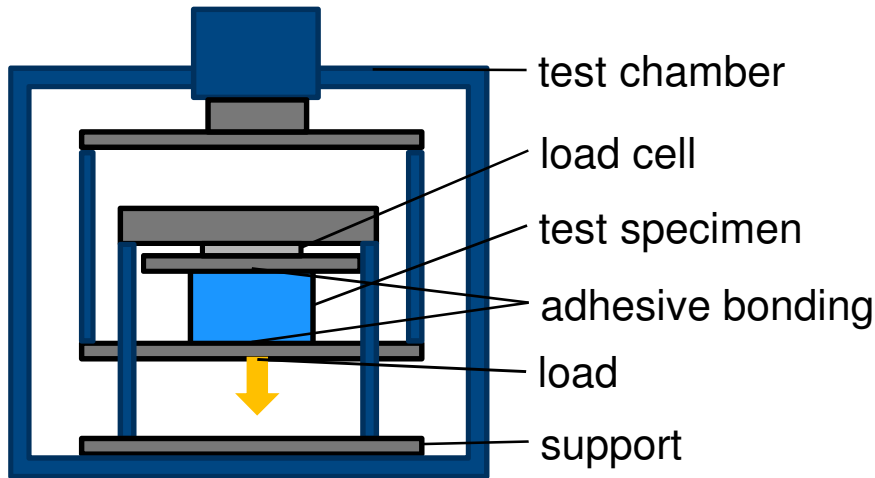


Outline

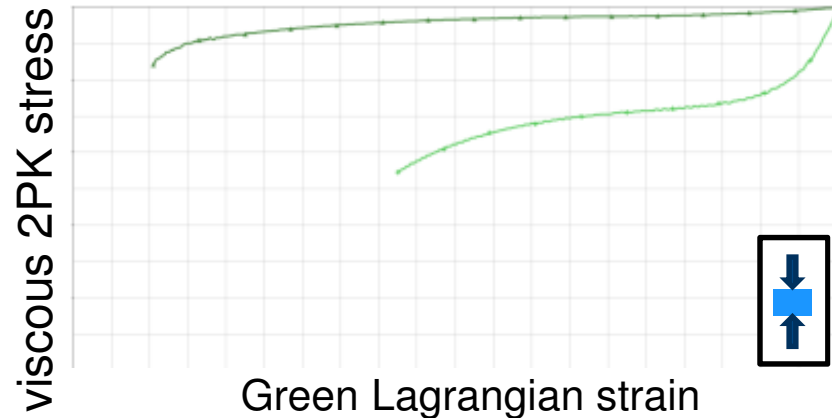
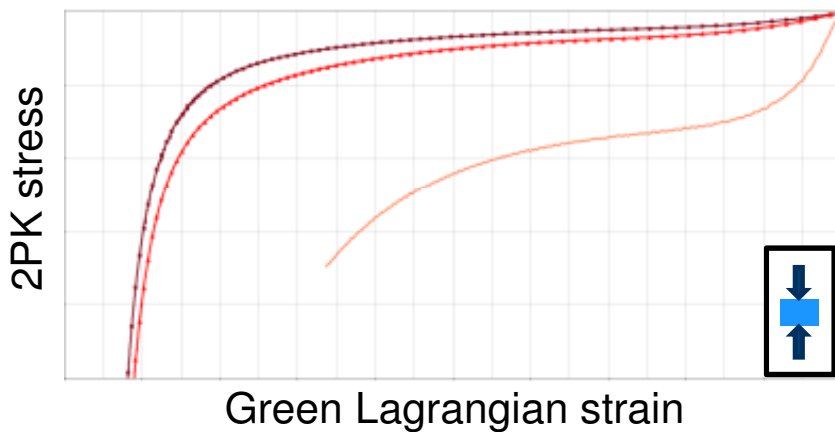
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Modeling of Confor[®] foam

- Testing in uniaxial tension and compression load

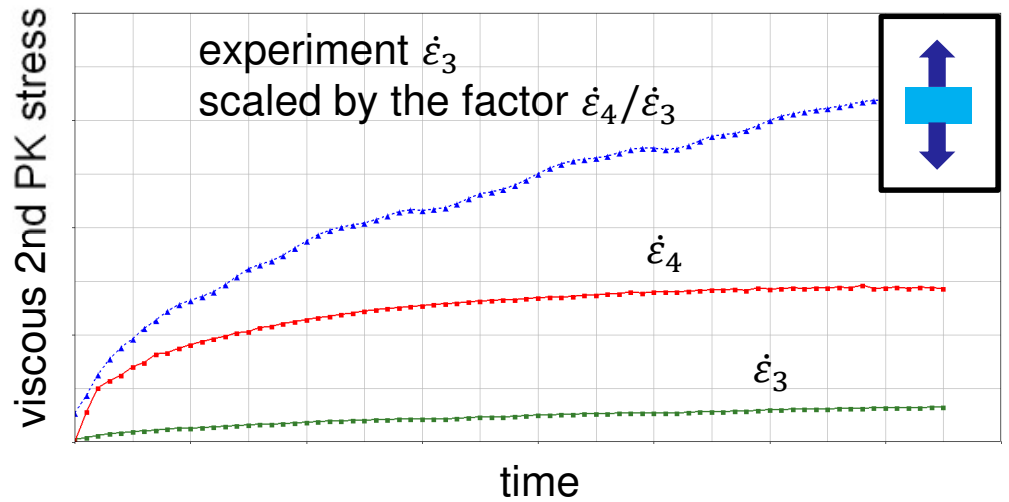
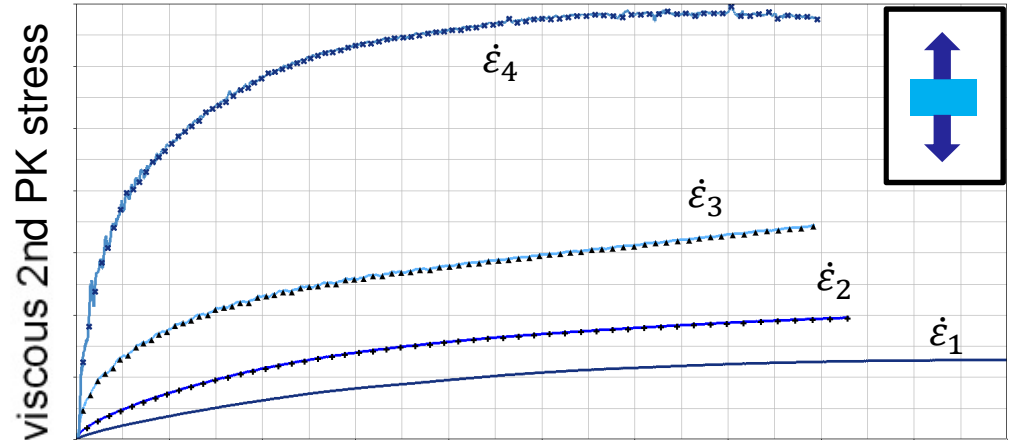
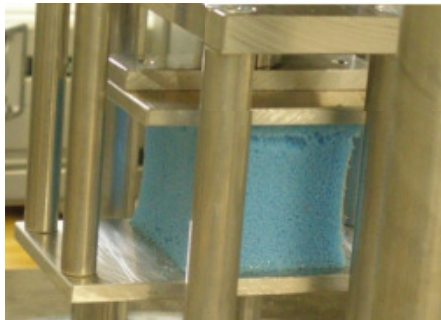
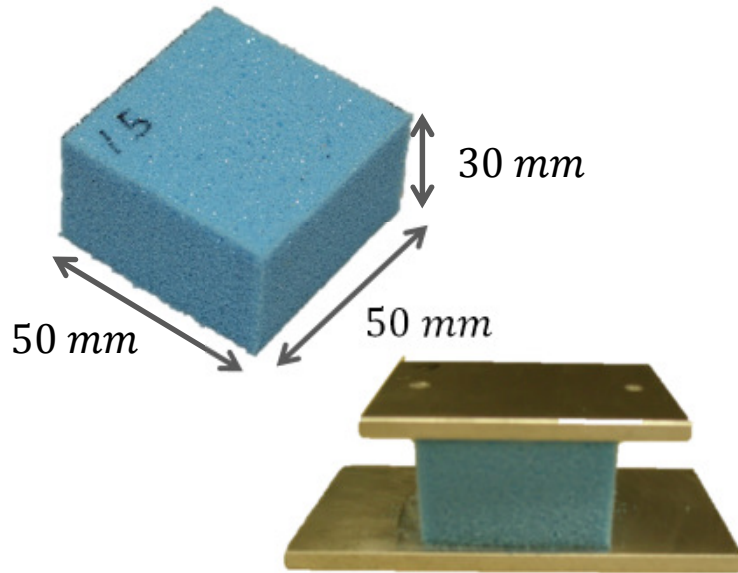


Viscous stress = Dynamic testing - Quasistatic testing



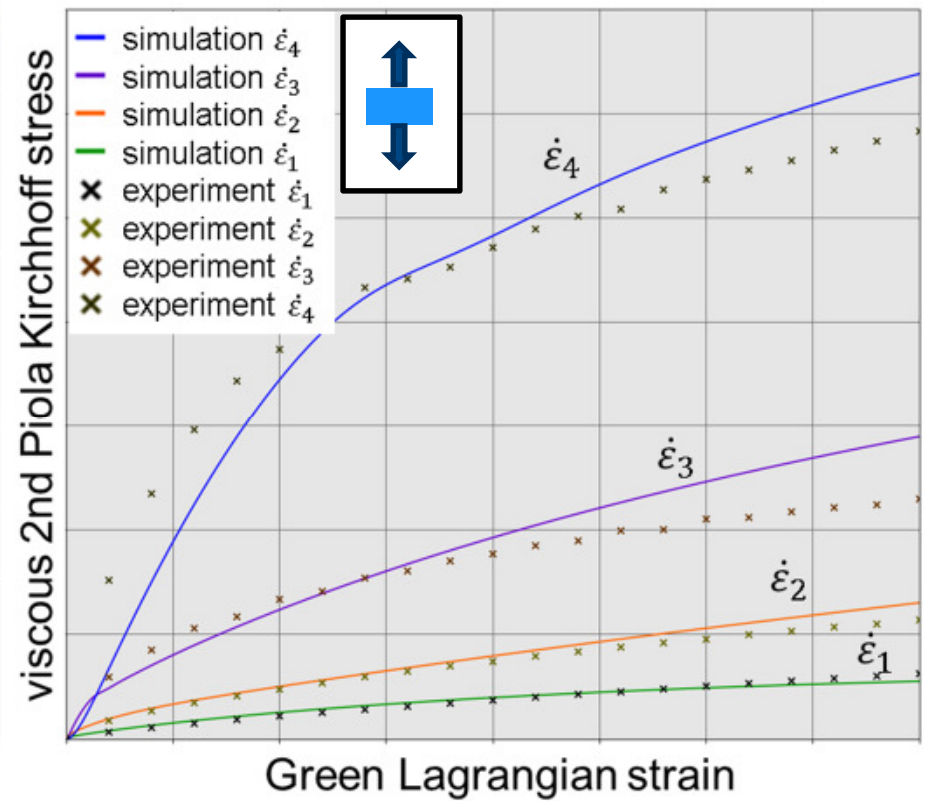
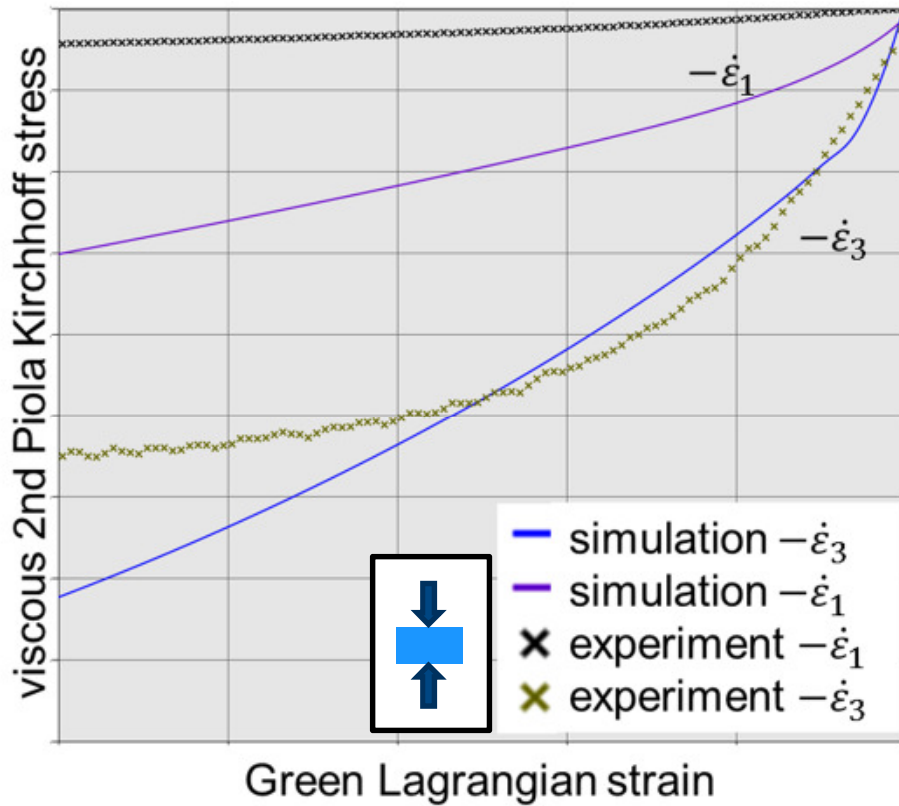
Modeling of Confor[®] foam

- Nonlinear viscoelastic properties



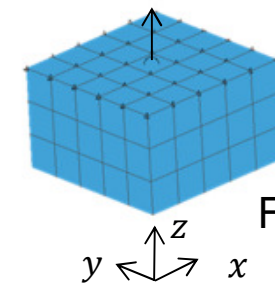
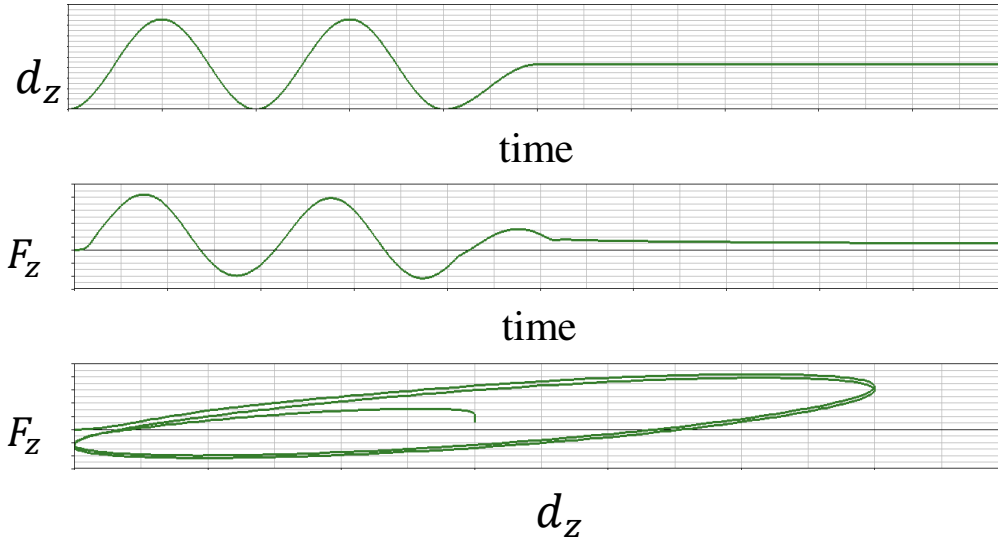
Modeling of Confor[®] foam

- Parameter identification up to 20% GL strain

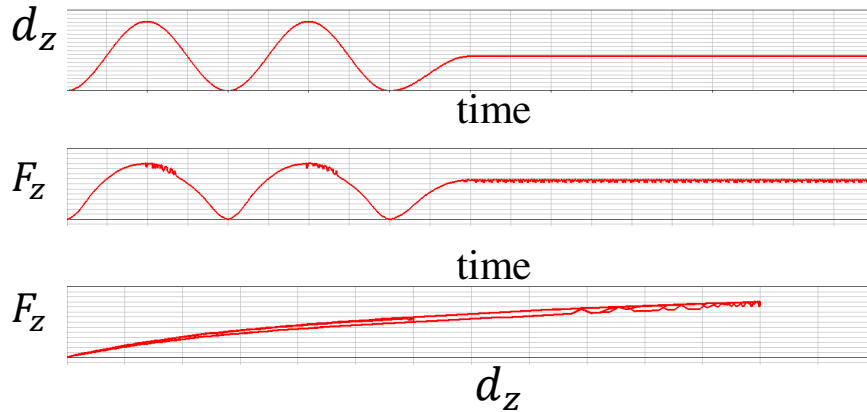


Modeling of Confor[®] foam

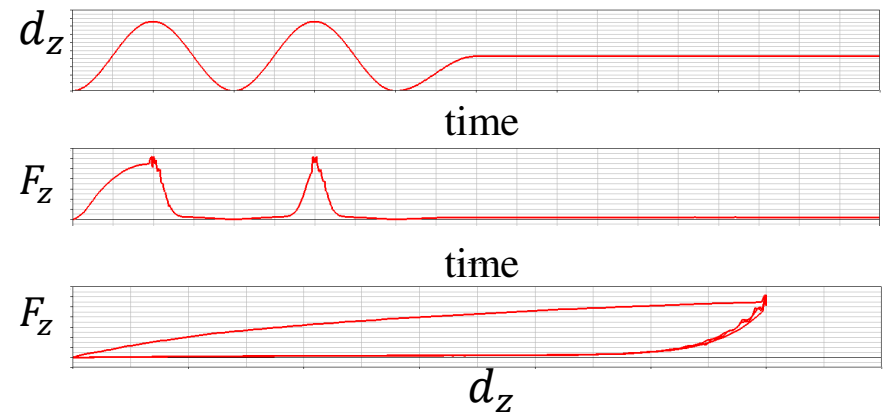
- Green-Rivlin model for foams



- *MAT_083 without damage



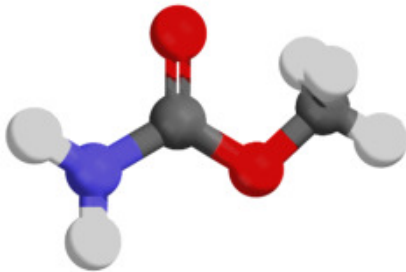
- *MAT_083 with damage



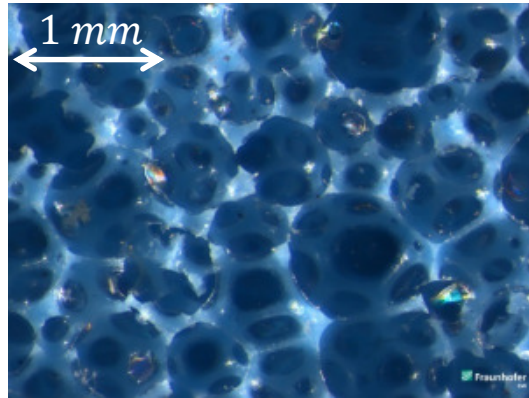
Modeling of Confor[®] foam

- Continuum approach to open-cell foams

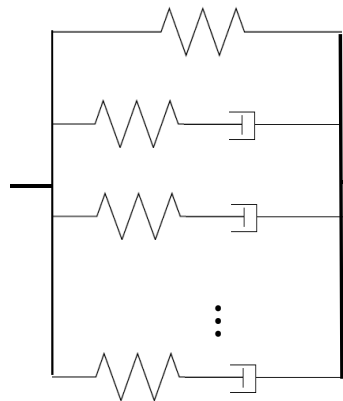
Material



Foam structure

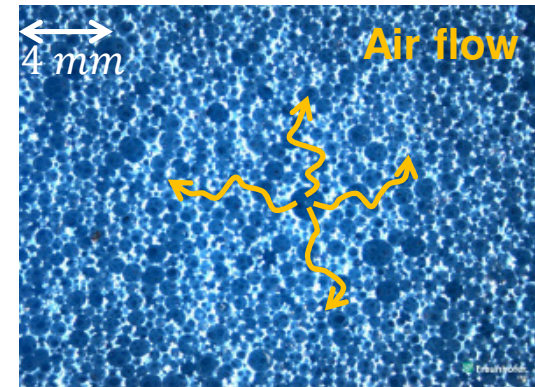


Sample size



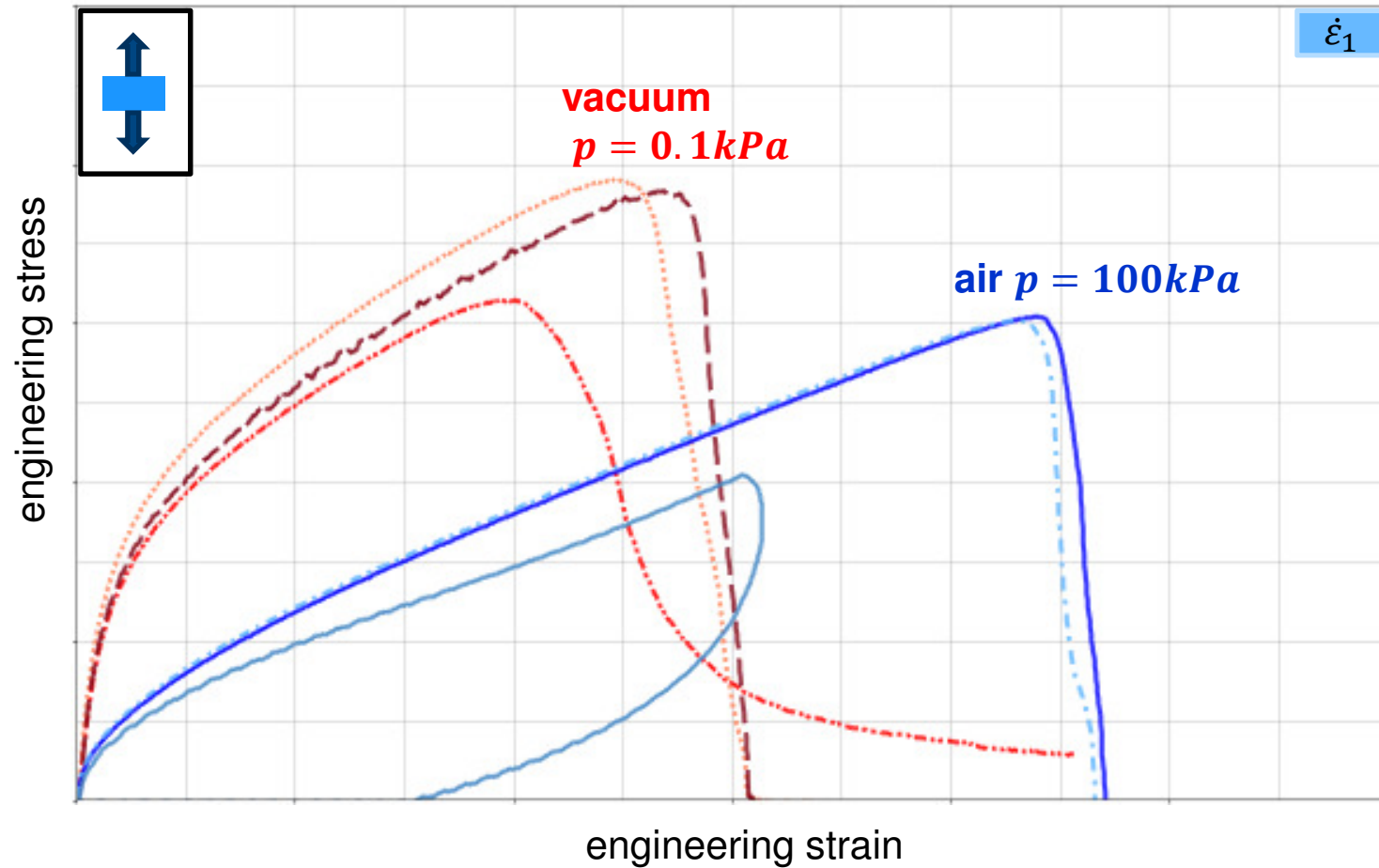
- ✓ Equilibrium material response: asymmetric behaviour under compression and tension
- ✓ Non-equilibrium material response: nonlinear viscoelasticity

Porous media: gas solid interaction



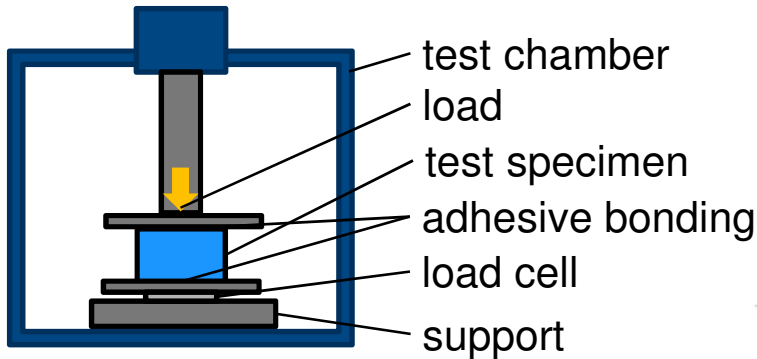
Modeling of Confor[®] foam

- Testing in air and vacuum



Modeling of Confor[®] foam

- Dynamic Mechanical Analysis (DMA): Glass transition temperature



Enforced displacement

$$d = d_0 \sin(\omega t)$$

Force measurement

$$\frac{F}{A_0} = \frac{F_0}{A_0} \sin(\omega t + \delta)$$

$$= |E^*| \varepsilon_0 \sin(\omega t + \delta)$$

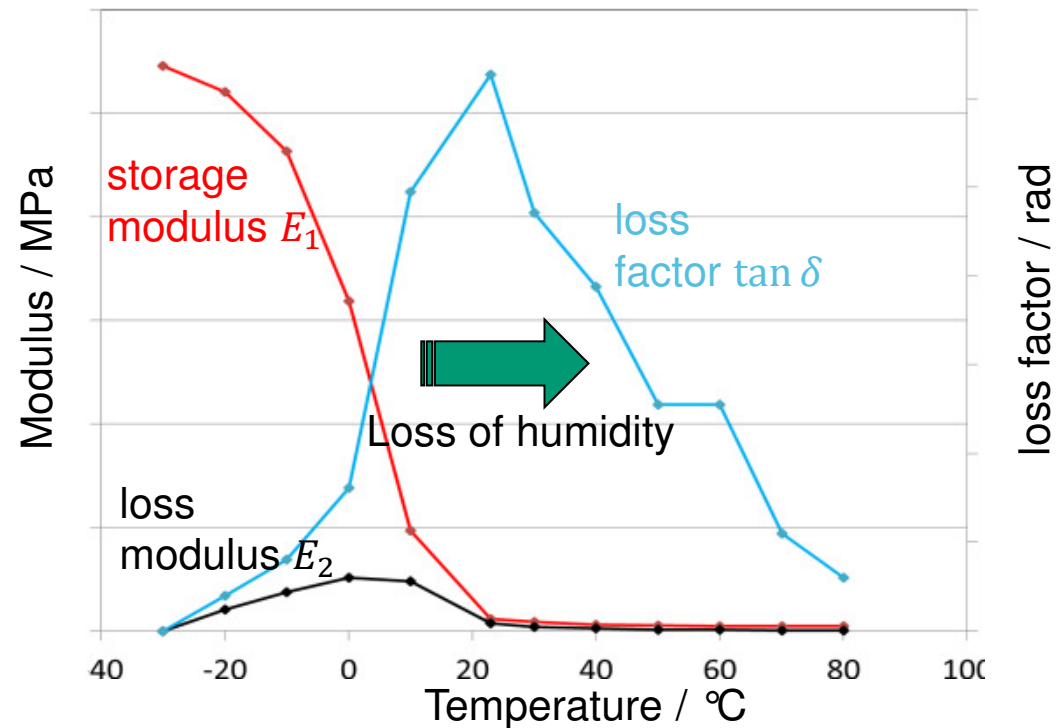
Complex modulus

$$E^* = |E^*| e^{i\delta}$$

$$= (E_1 + iE_2)$$

Precompression: 30% eng. Strain

$$d_0 = 0.1 \text{ mm}, f = 10 \text{ 1/s}$$





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Summary

- State of the art modeling without viscoelasticity
- Limits of linear viscoelasticity
- Improvement with nonlinear viscoelastic constitutive equation for foams
- Further research on parameter identification
- No separation of viscoelasticity and air flow with vacuum testing for Confor foam

Literature

- Effinger, V. et. al.: “Nonlinear Viscoelastic Modeling for Foams”, 13th International LS-DYNA Users Conference Detroit, 2014
<http://www.dynalook.com/13th-international-ls-dyna-conference/constitutive-modeling/nonlinear-viscoelastic-modeling-for-foams>