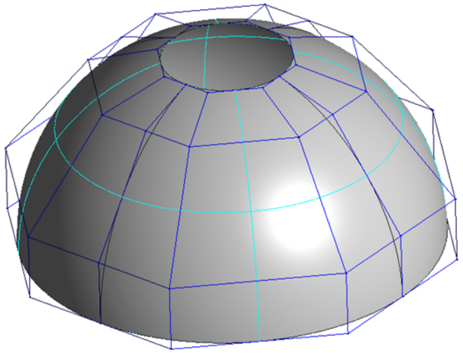
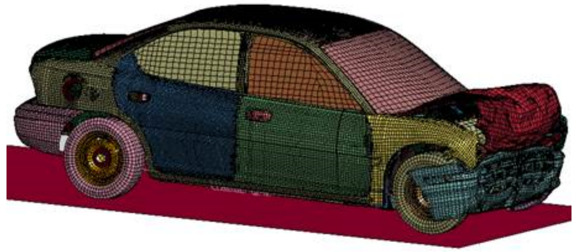


LS-DYNA Forum 2014

7 October 2014
Bamberg



On two Recent Advances in Computational Mechanics

Isogeometric Analysis of Shells and Variational Mass Scaling

Manfred Bischoff,
Ralph Echter, Bastian Oesterle, Martina Matzen, Ekkehard Ramm,
Anton Tkachuk, Anne Schäuble

Universität Stuttgart
Germany

Baustatik und Baudynamik

Isogeometric Analysis

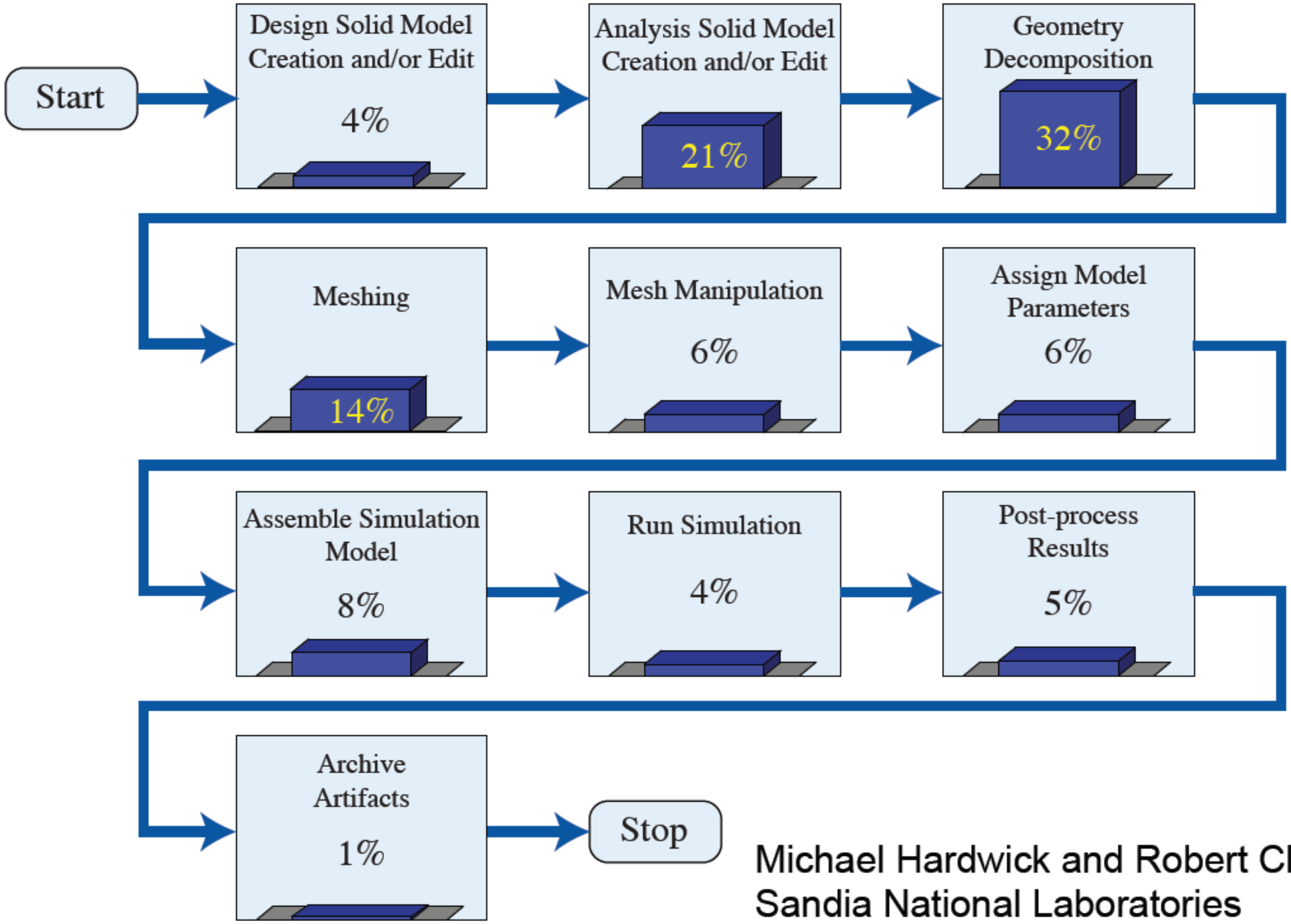
- Basic Idea
- Isogeometric Finite Elements and Locking
- Isogeometric Shell Elements
- Numerical Examples

Variational Mass Scaling

- Motivation
- Penalized Hamilton's Principle
- Discretization
- Numerical Examples

typical steps of computational analysis procedure

from design model to analysis results (CAD and FEA)

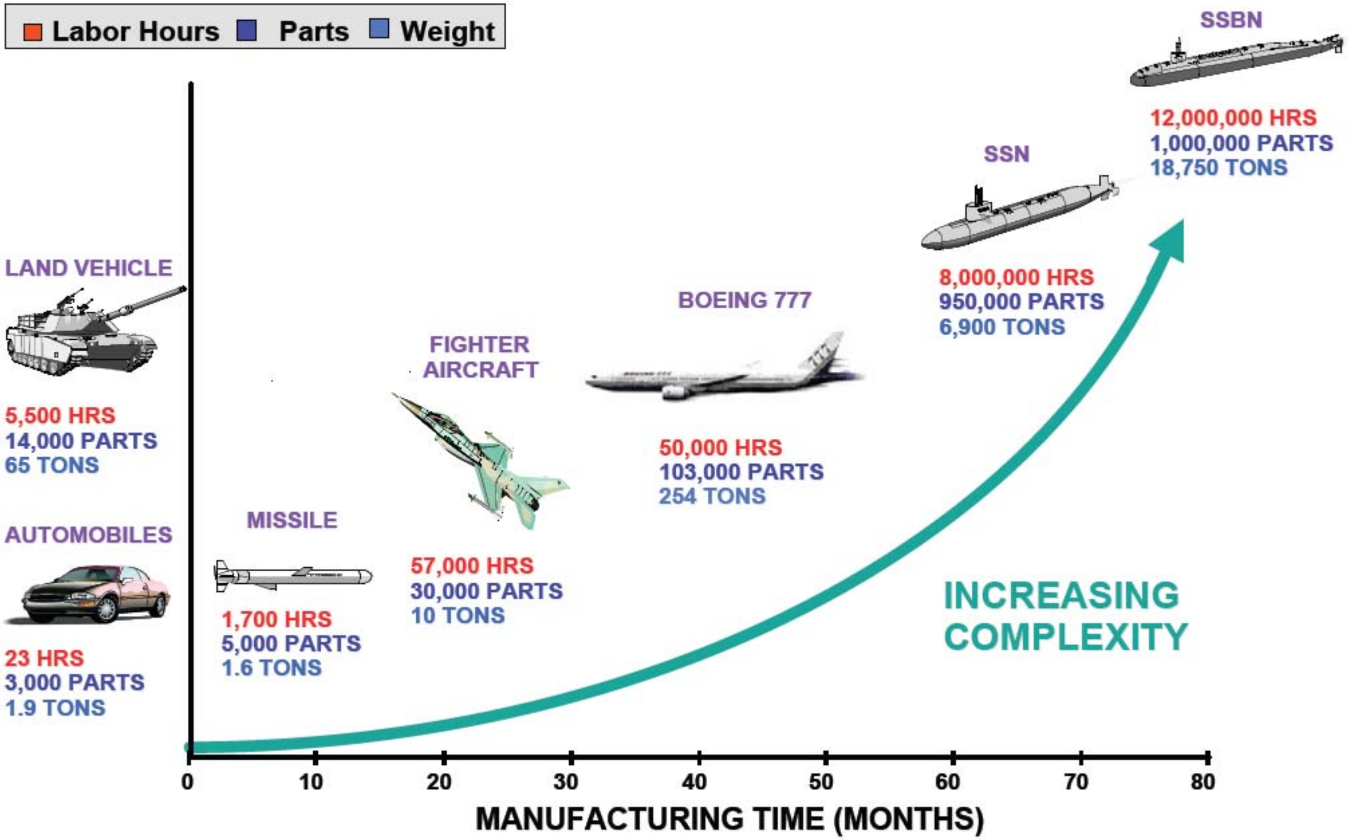


Michael Hardwick and Robert Clay,
Sandia National Laboratories

HUGHES, COTTRELL, BAZILEVS. ISOGEOMETRIC ANALYSIS: CAD, FINITE ELEMENTS, NURBS, EXACT GEOMETRY AND MESH REFINEMENT. COMPUTER METHODS IN APPLIED MECHANICS AND ENGINEERING 194, 4135-4195, 2005.

typical steps of computational analysis procedure

from design model to analysis results (CAD and FEA)

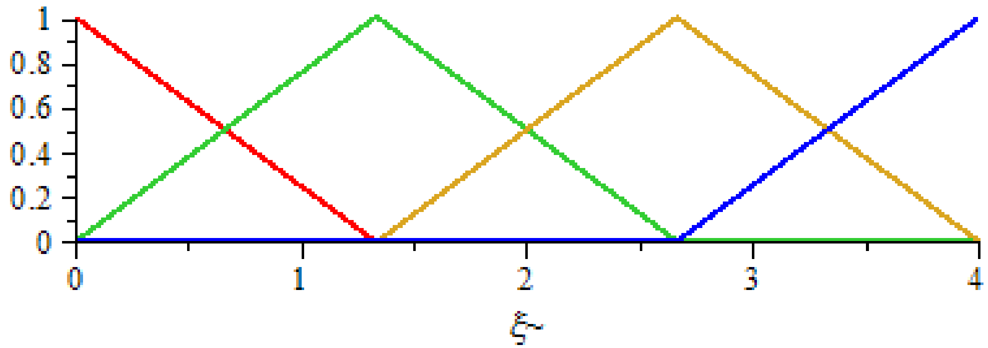


Courtesy of General Dynamics / Electric Boat Corporation

HUGHES, COTTRELL, BAZILEVS. ISOGEOMETRIC ANALYSIS: CAD, FINITE ELEMENTS, NURBS, EXACT GEOMETRY AND MESH REFINEMENT. COMPUTER METHODS IN APPLIED MECHANICS AND ENGINEERING 194, 4135-4195, 2005.

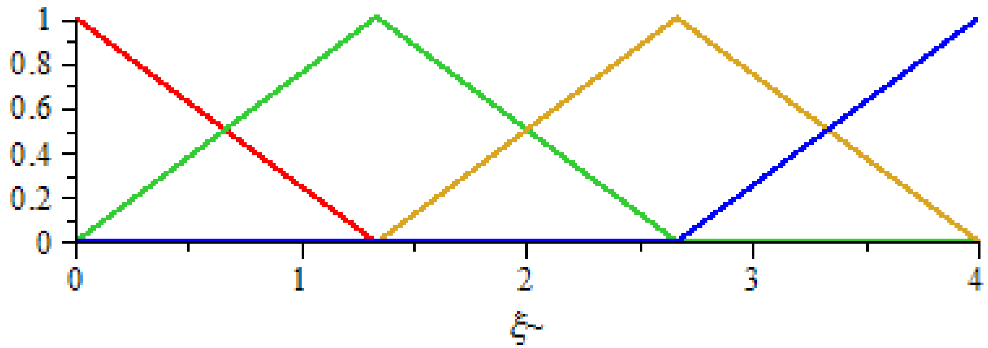
finite elements formulation with CAD parameterization

standard FEM discretization space (one-dimensional): 3 elements, p = 1



C^0-continuous

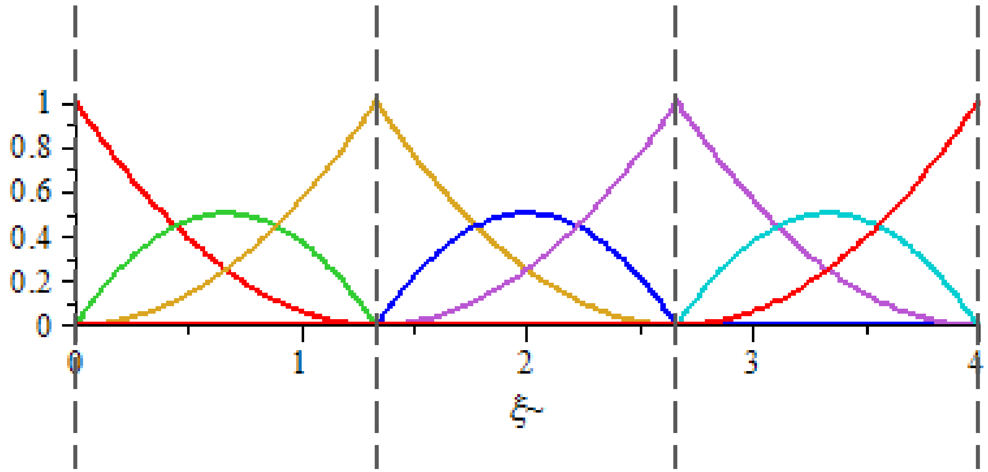
B-splines or NURBS (non-uniform rational B-splines): 3 elements, p = 1



C^0-continuous

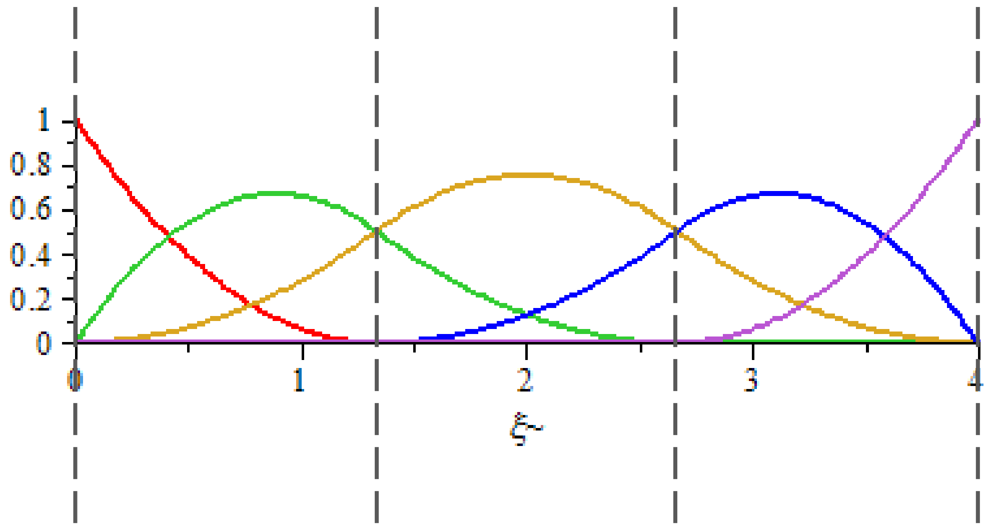
finite elements formulation with CAD parameterization

standard FEM discretization space (one-dimensional): 3 elements, p = 2



C^0-continuous

B-splines or NURBS (non-uniform rational B-splines): 3 elements, p = 2



C^1-continuous

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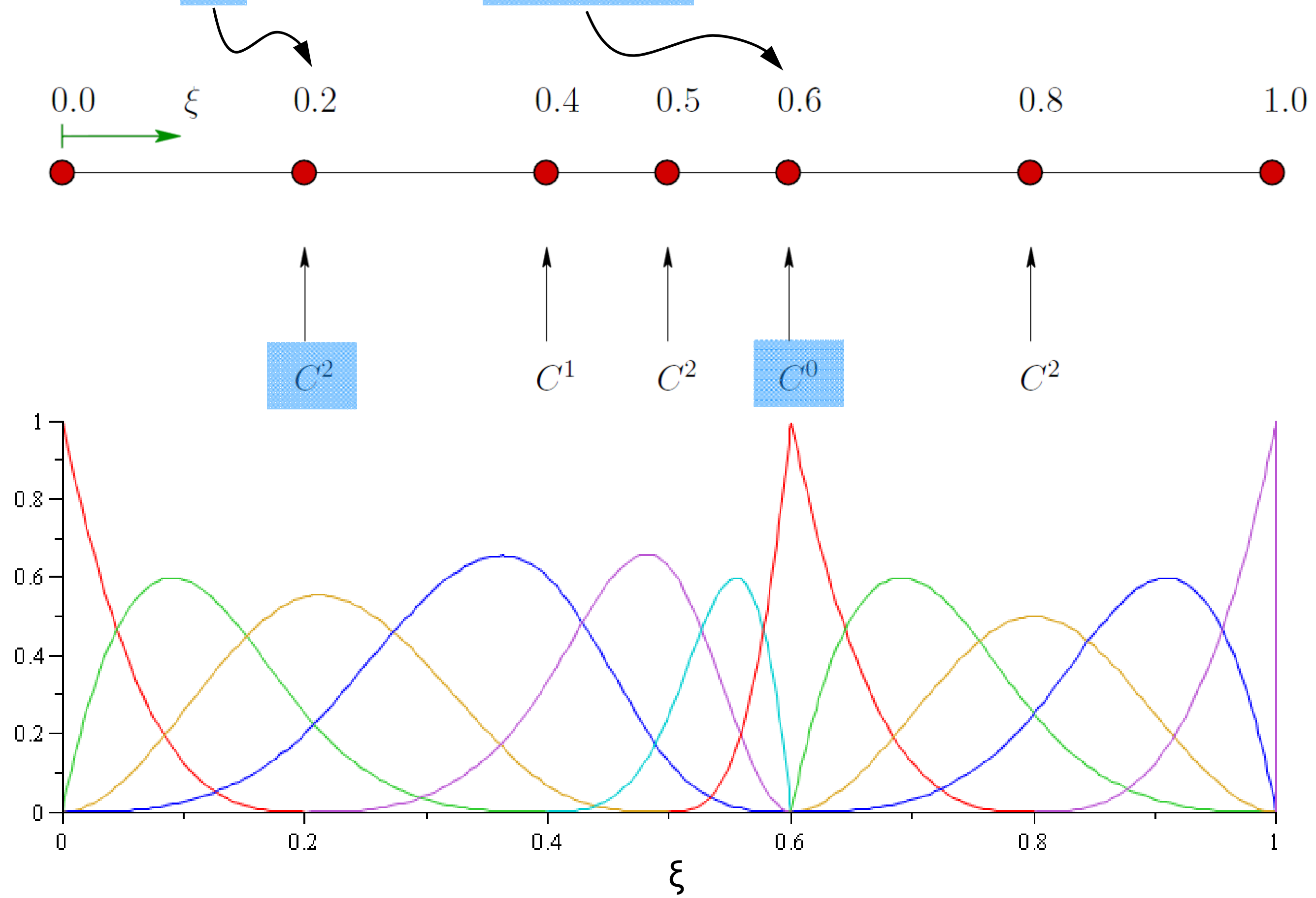
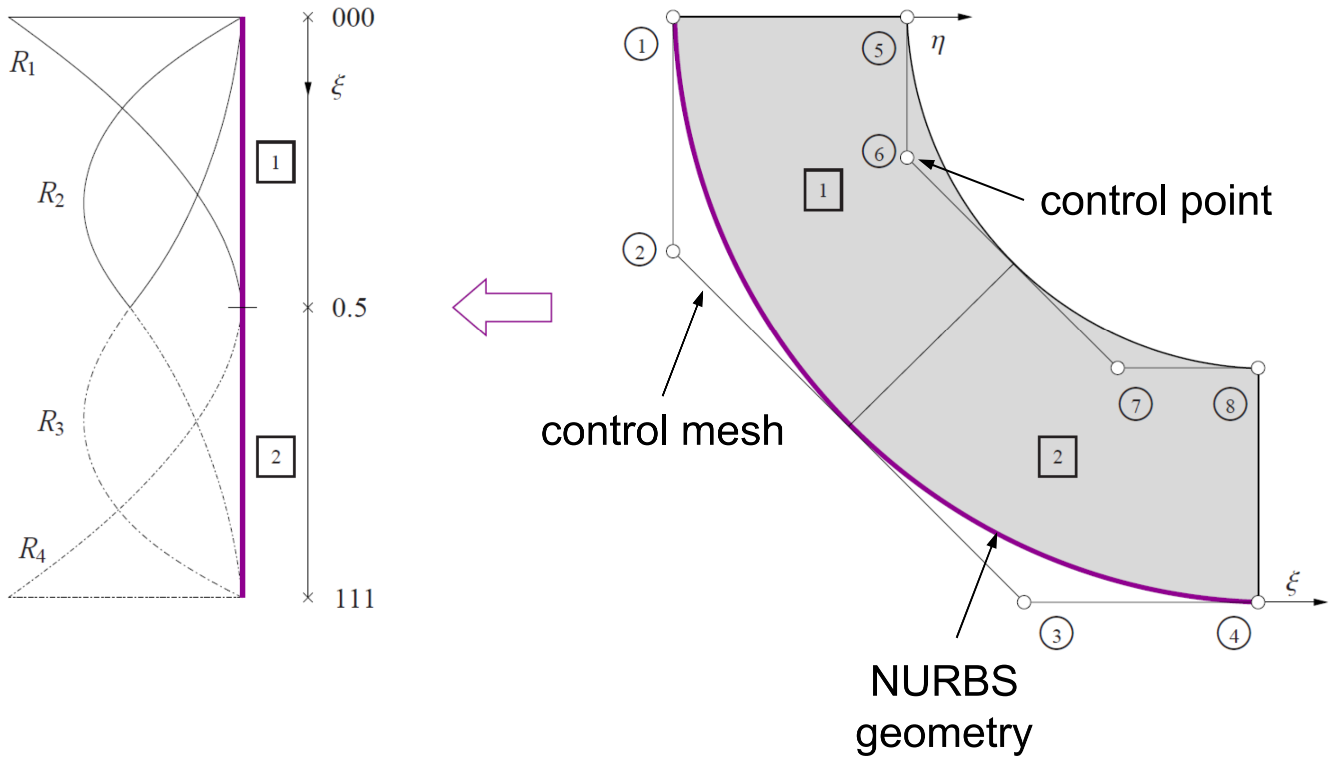
knot vector and control points

maximum continuity: C^{p-1}

$$\Xi = [0, 0, 0, 0, 0.2, 0.4, 0.4, 0.5, 0.6, 0.6, 0.6, 0.8, 1, 1, 1, 1]$$

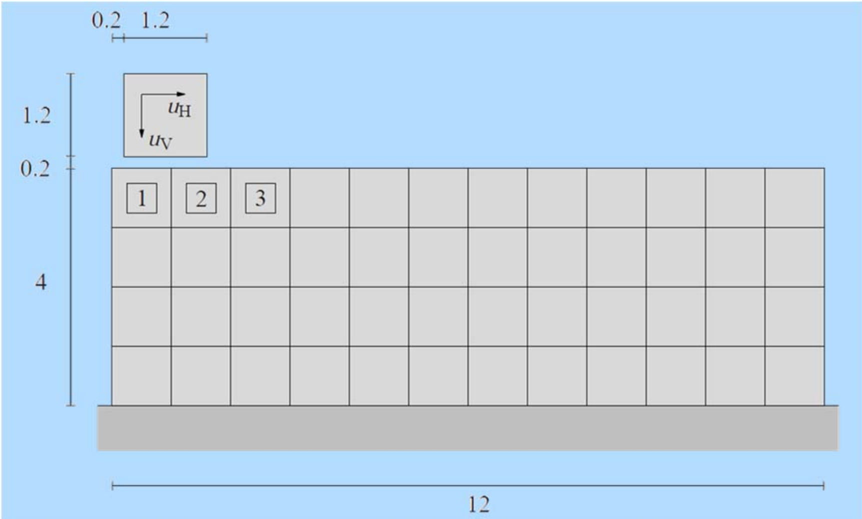


geometry description with NURBS



- control points (cf. FEM nodes) are not interpolatory
- degrees of freedom (e.g. displacements) are not interpolatory

example: large sliding contact, ironing problem

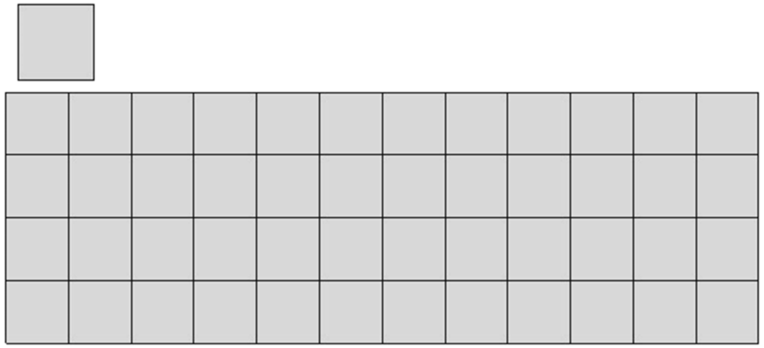


length in mm

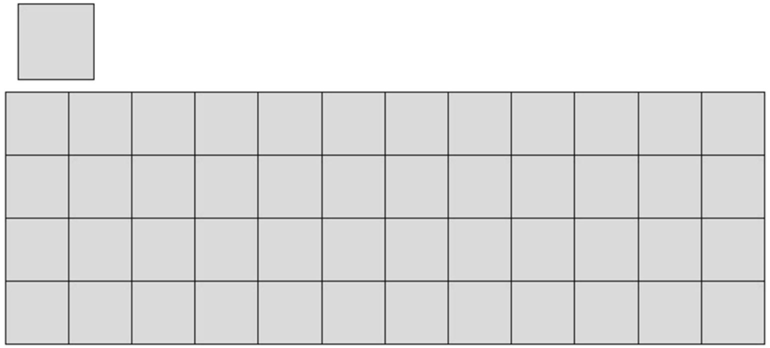
displacement control:
u_V = 0.8 mm, u_H = 10.5 mm

St. Venant-Kirchhoff, plane stress
ν = 0.32
$E_1 = 68.96 \cdot 10^8$ N/mm^2
$E_2 = 68.96 \cdot 10^7$ N/mm^2
p = 3

Lagrange discretization



NURBS discretization



arrows: Lagrange multipliers = contact forces at collocation points

MATZEN, CICHOSZ, BISCHOFF. A POINT TO SGMENT CONTACT FORMULATION FOR ISOGEOMETRIC, NURBS BASED FINITE ELEMENTS. COMPUTER METHODS IN APPLIED MECHANICS AND ENGINEERING 255, 27-39, 2013.

finite elements for shells

Kirchhoff-Love (3-parameter model) — **requires C^1-continuity**

Reissner-Mindlin (5-parameter model)

3d-shell (7-parameter model)

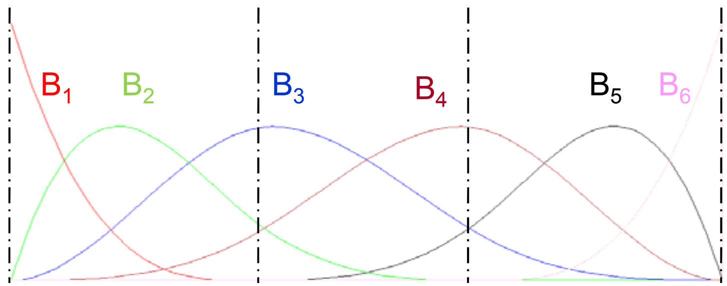
} **C^0-continuous FEM**

isogeometric analysis and finite elements

isoparametric approach + exact geometry representation

here: B-spline and NURBS basis functions

higher continuity within patches (C^1, C^2,…, C^{p-1})

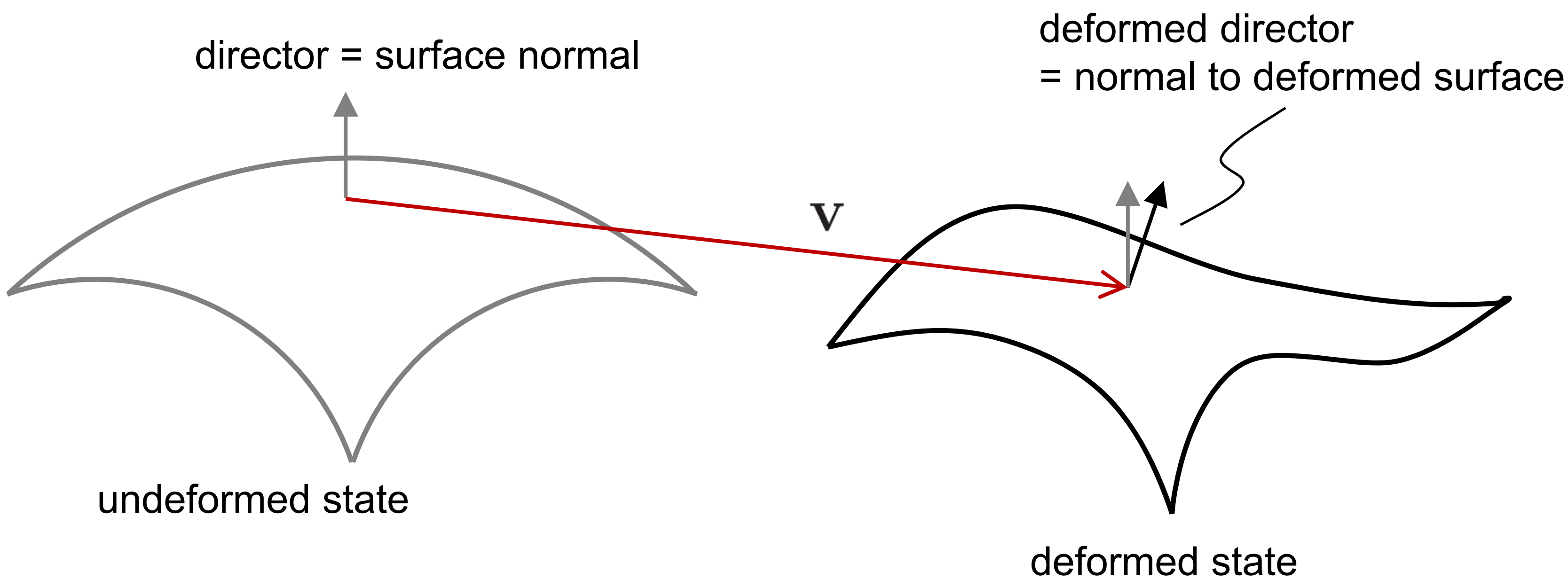


KIENDL, BLETZINGER, LINHARD, WÜCHNER. ISOGEOMETRIC SHELL ANALYSIS WITH KIRCHHOFF-LOVE ELEMENTS. COMPUTER METHODS IN APPLIED MECHANICS AND ENGINEERING 198, 3902-3914, 2009.

ECHTER, OESTERLE, BISCHOFF. A HIERARCHIC FAMILY OF ISOGEOMETRIC SHELL FINITE ELEMENTS. COMPUTER METHODS IN APPLIED MECHANICS AND ENGINEERING 254, 170-180, 2013.

RALPH ECHTER. ISOGEOMETRIC ANALYSIS OF SHELLS. PHD DISSERTATION, INSTITUT FÜR BAUSTATIK UND BAUDYNAMIK, UNIVERSITÄT STUTTGART, 2013.

Kirchhoff-Love

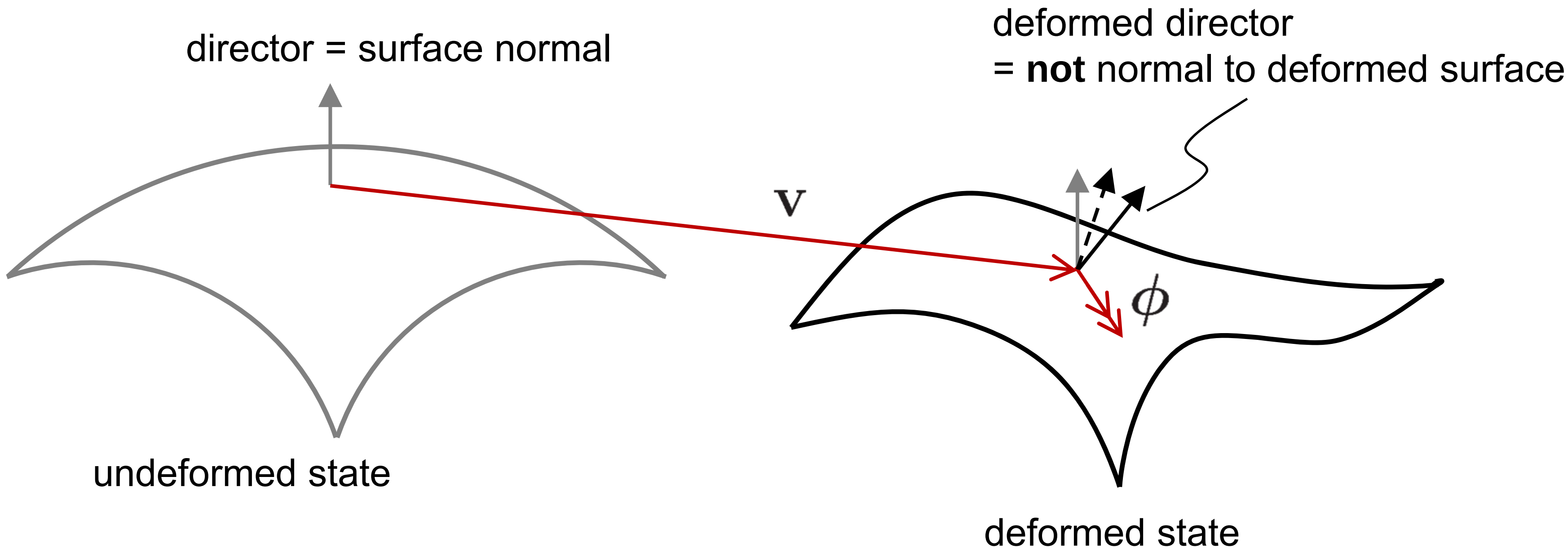


3-parameter model

3 d.o.f. = 3 components of displacement vector **v**

uniqueness of rotated normal
requires C^1-continuity

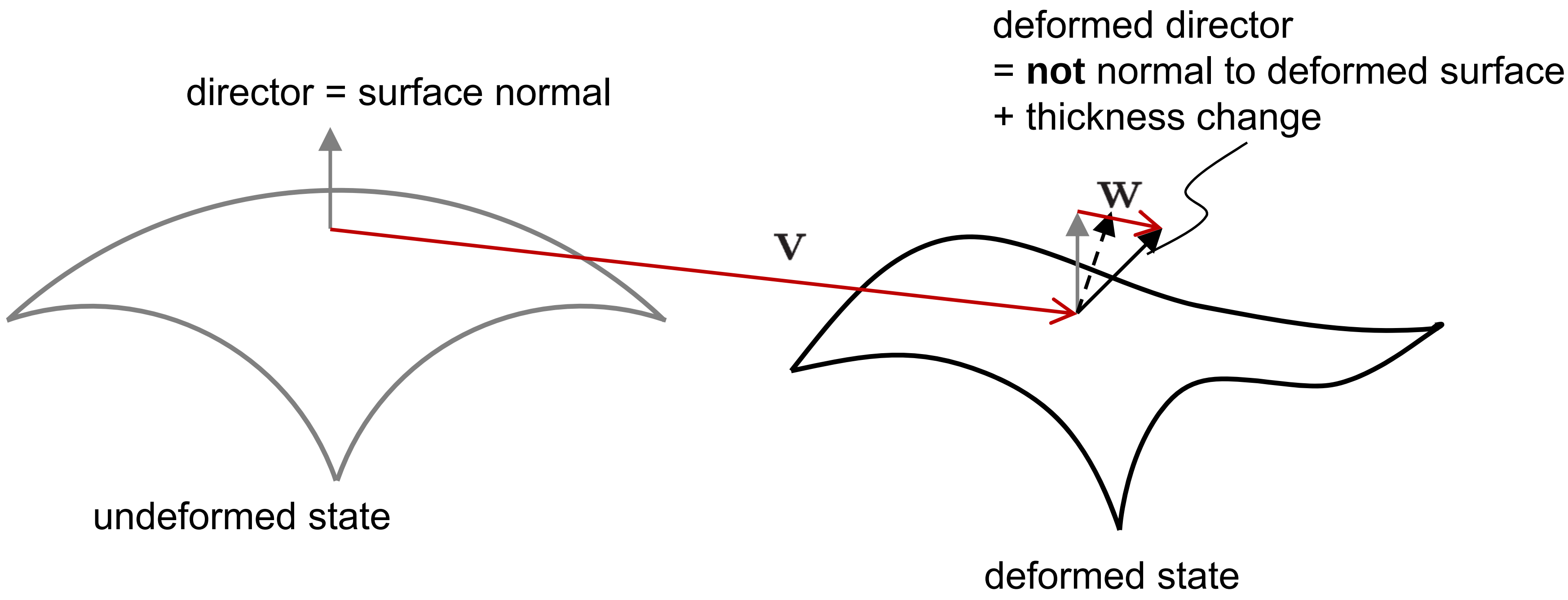
Reissner-Mindlin



5-parameter model

5 d.o.f. = 3 components of displacement vector **v**
2 independent rotations Φ

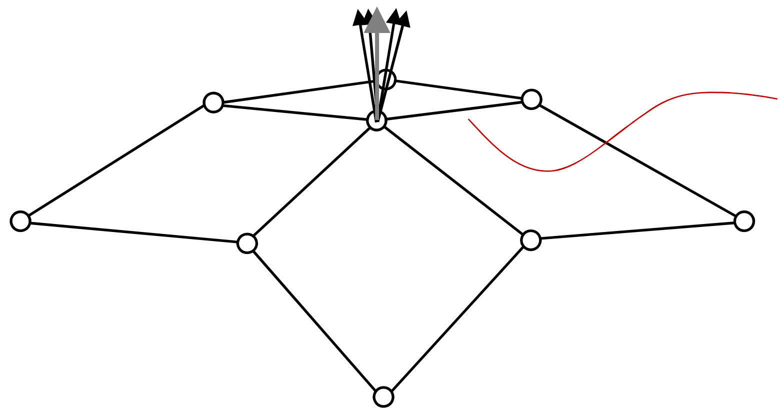
3d-shell



7-parameter model

7 d.o.f. = 3 components of displacement vector **v**
3 components of difference vector **w**
1 linear transverse normal strain component ε_{33}

issues



ambiguous definition of director

artificial drilling rotations needed (standard approach in commercial codes)

alternative: average nodal director (violates normality condition)

more issues

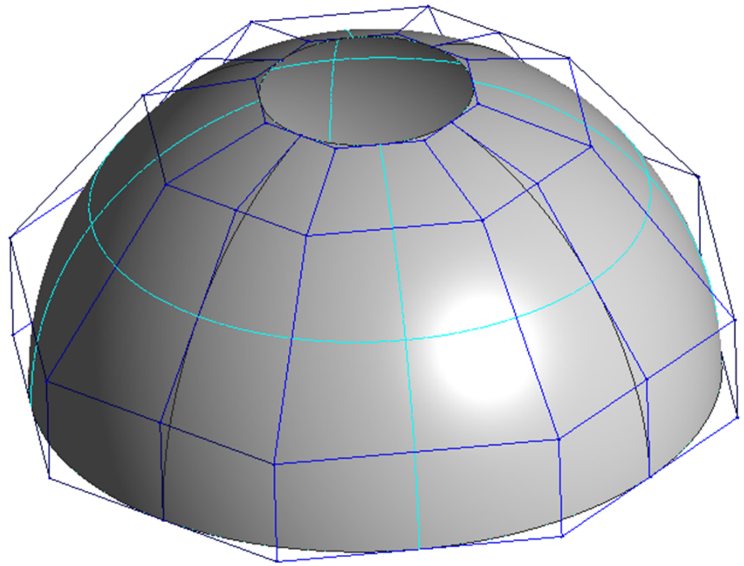
notorious locking phenomena

1,000,000 papers and counting

known benefits from isogeometric analysis

higher continuity facilitates Kirchhoff-Love elements (KIENDL ET AL. [2009])

higher continuity improves finite element approximation (ECHTER AND BISCHOFF [2010])



potential for more benefits

C^1-continuous formulation also for Reissner-Mindlin and 3d-shells

hierarchic family of 3p-, 5p- and 7p-shells

$\rightarrow$ model adaption, hybrid models, unique implementation

standard displacement formulation

cf. KIENDL ET AL. [2009]

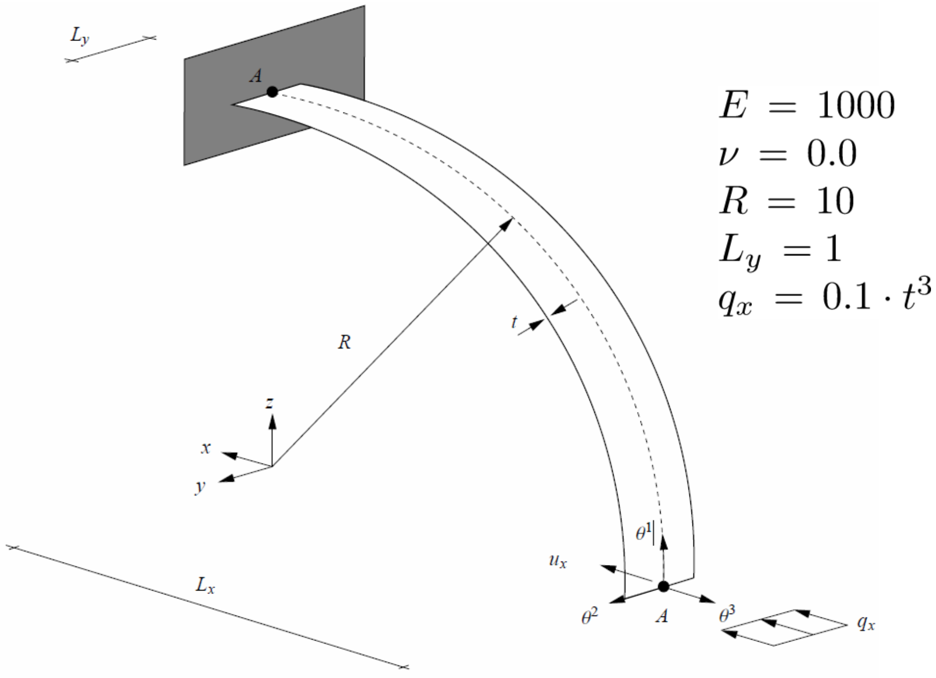
our implementation uses stresses instead of stress resultants

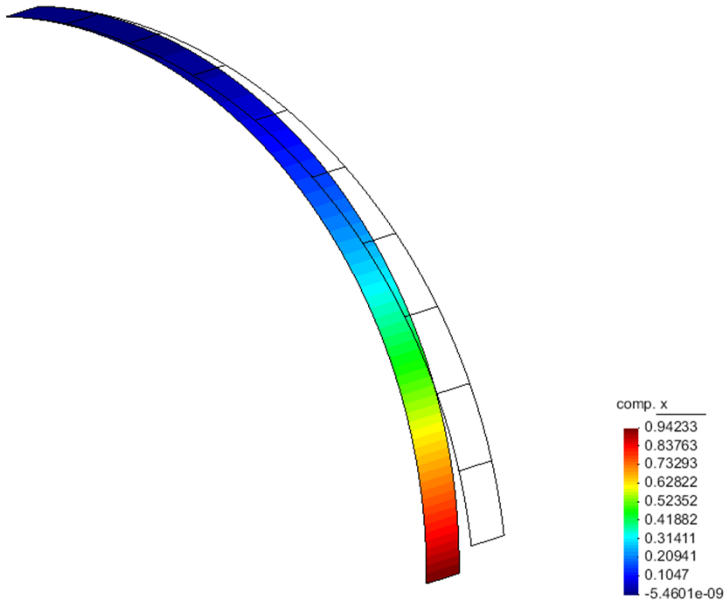
numerical experiment: cylindrical shell strip

2nd, 3rd, 4th and 5th order shape functions

varying thickness (thick to very thin)

L_2-norm of displacement error





Numerical Experiment: Cylindrical Shell Strip

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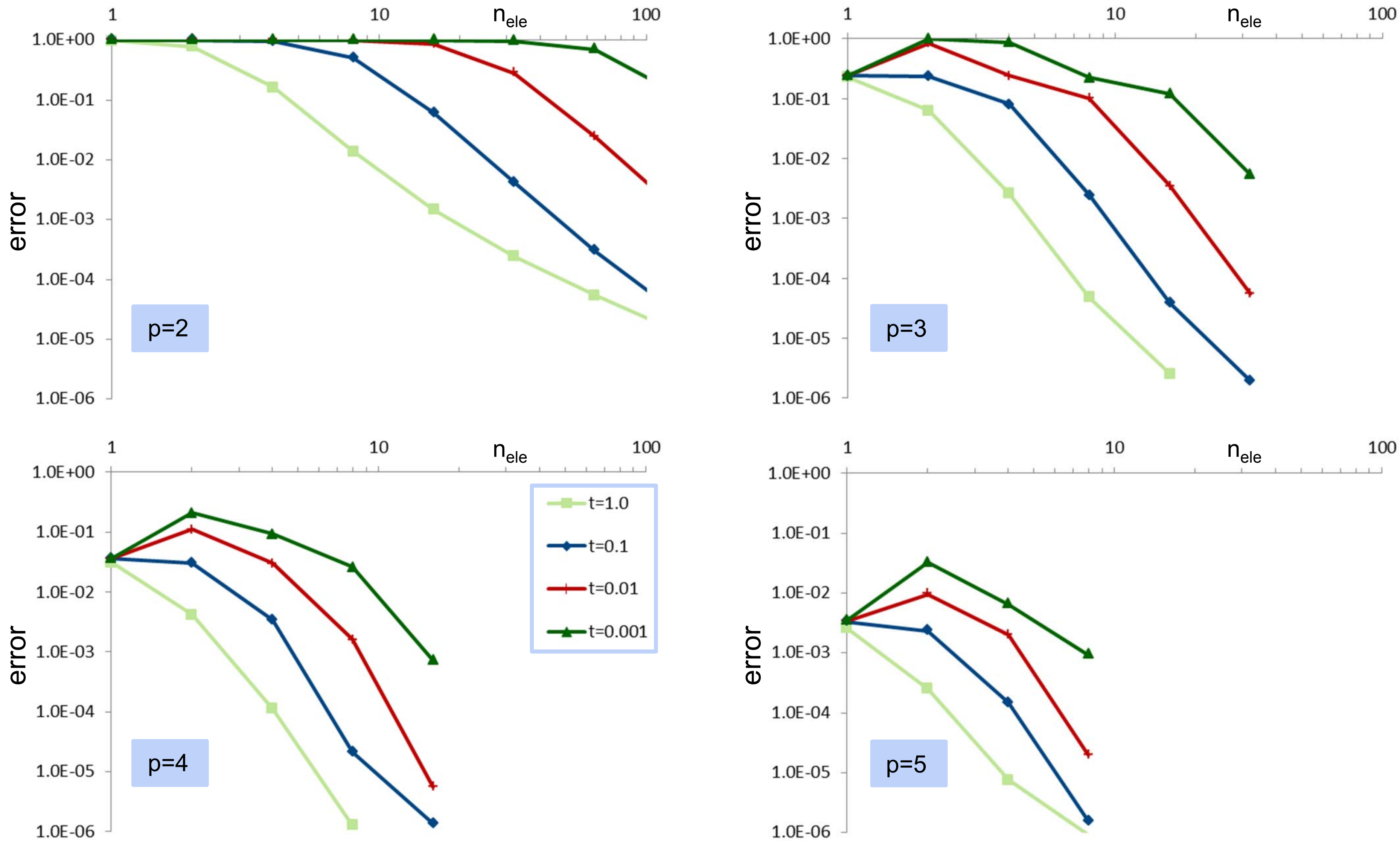
L_2-norm of error in displacements **membrane locking – all polynomial orders!**



next steps

NURBS-based shell formulation

hierarchic family of 3p-, 5p- and 7-models

removing locking

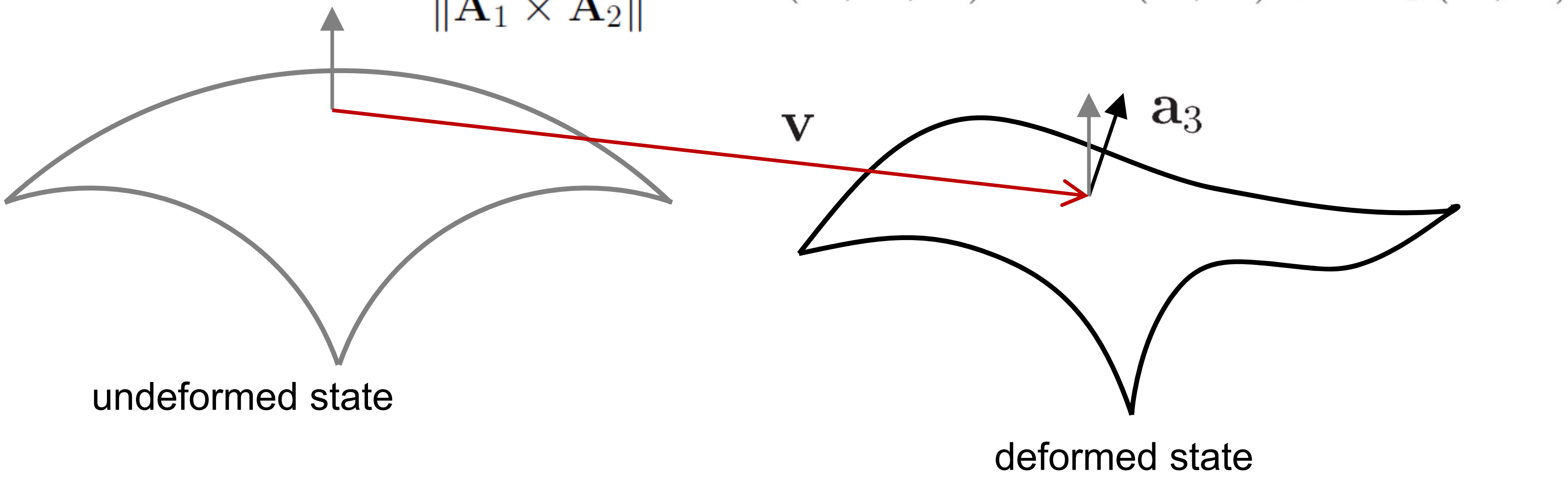
membrane locking (3p, 5p, 7p)

+ transverse shear locking (5p, 7p)

+ curvature thickness locking (7p)

3-parameter model (Kirchhoff-Love)

$$\mathbf{A}_3 = \frac{\mathbf{A}_1 \times \mathbf{A}_2}{\|\mathbf{A}_1 \times \mathbf{A}_2\|} \qquad \mathbf{x}\left(\theta^1, \theta^2, \theta^3\right) = \mathbf{r}\left(\theta^1, \theta^2\right) + \theta^3\, \mathbf{a}_3\left(\theta^1, \theta^2\right)$$



computation of deformed director

$$\mathbf{a}_3 = \mathbf{A}_3 + \mathbf{e} \times \mathbf{A}_3$$

$$\mathbf{e} = \varphi_1 \mathbf{A}_1 + \varphi_2 \mathbf{A}_2$$

linearized rotations:

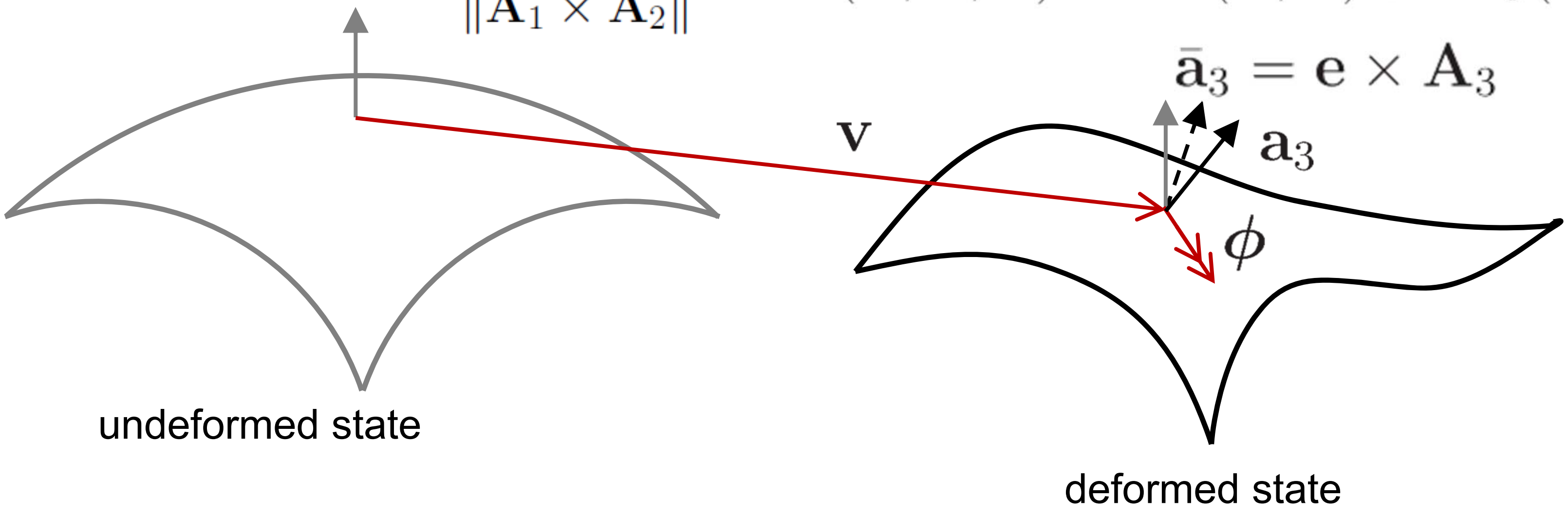
$$\varphi_1 = (\mathbf{a}_2 - \mathbf{A}_2) \cdot \mathbf{A}_3 = \mathbf{v}_{,2} \cdot \mathbf{A}_3$$

$$\varphi_2 = -(\mathbf{a}_1 - \mathbf{A}_1) \cdot \mathbf{A}_3 = -\mathbf{v}_{,1} \cdot \mathbf{A}_3$$

5-parameter model (Reissner-Mindlin)

$$\mathbf{A}_3 = \frac{\mathbf{A}_1 \times \mathbf{A}_2}{\|\mathbf{A}_1 \times \mathbf{A}_2\|} \qquad \mathbf{x}\left(\theta^1, \theta^2, \theta^3\right) = \mathbf{r}\left(\theta^1, \theta^2\right) + \theta^3 \, \mathbf{a}_3 \left(\theta^1, \theta^2\right)$$

$$\bar{\mathbf{a}}_3 = \mathbf{e} \times \mathbf{A}_3$$



computation of deformed director

$$\mathbf{a}_3 = \mathbf{A}_3 + \boldsymbol{\phi} \times \mathbf{A}_3$$

hierarchic rotation

hierarchic concept*:

$$\mathbf{a}_3 = \mathbf{A}_3 + \bar{\boldsymbol{\phi}} \times \bar{\mathbf{a}}_3 = \mathbf{A}_3 + \bar{\boldsymbol{\phi}} \times (\mathbf{A}_3 + \mathbf{e} \times \mathbf{A}_3)$$

*BASAR, KRÄTZIG [1985], LONG, BORNEMANN, CIRAK [2011]

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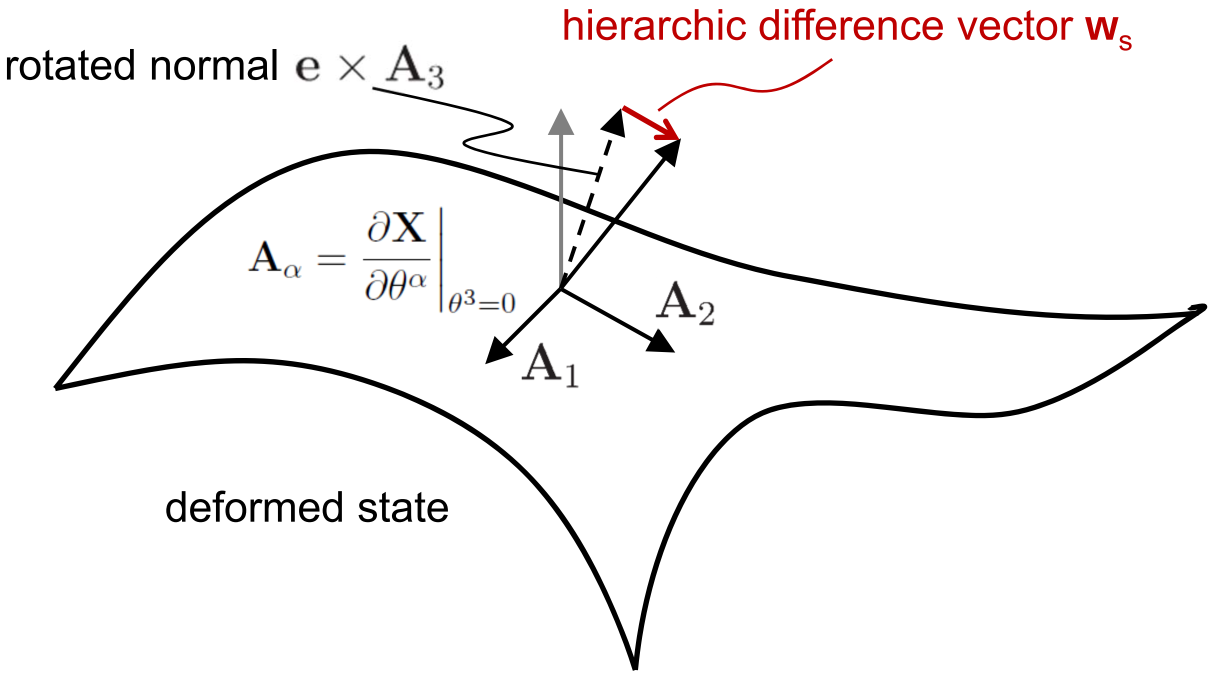
5-parameter model (Reissner-Mindlin)



computation of deformed director

$$\mathbf{a}_3 = \mathbf{A}_3 + \underline{\mathbf{e} \times \mathbf{A}_3} + \mathbf{w}_s$$

Kirchhoff-Love

+ transverse shear

$$\mathbf{w}_s = w^\alpha \cdot \mathbf{A}_\alpha$$

2 d.o.f.
inextensibility condition
naturally included

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5-parameter model (Reissner-Mindlin)

displacements

$$\begin{aligned}\mathbf{u} &= \mathbf{x} - \mathbf{X} \\ &= \mathbf{r} + \theta^3\,\mathbf{a}_3 - \mathbf{R} - \theta^3\mathbf{A}_3 \\ &= \mathbf{v} + \theta^3\,(\mathbf{a}_3 - \mathbf{A}_3) \\ &= \mathbf{v} + \theta^3\,(\mathbf{e} \times \mathbf{A}_3 + \mathbf{w}_s)\end{aligned}$$

linearized strains $\boldsymbol{\varepsilon} = \varepsilon_{ij}\,\mathbf{G}^i \otimes \mathbf{G}^j$ with $\varepsilon_{ij} = \frac{1}{2}(\mathbf{u}_{,i} \cdot \mathbf{G}_j + \mathbf{u}_{,j} \cdot \mathbf{G}_i)$

$$\mathbf{u}_{,\alpha} = \mathbf{v}_{,\alpha} + \theta^3\,(\mathbf{e}_{,\alpha} \times \mathbf{A}_3 + \mathbf{e} \times \mathbf{A}_{3,\alpha} + \mathbf{w}_{s,\alpha})$$

$$\mathbf{u}_{,3} = \mathbf{a}_3 - \mathbf{A}_3 = \mathbf{e} \times \mathbf{A}_3 + \mathbf{w}_s$$

$$\begin{aligned}\mathbf{G}_\alpha &= \frac{\partial \mathbf{X}}{\partial \theta^\alpha} = \mathbf{R}_{,\alpha} + \theta^3\mathbf{A}_{3,\alpha} = \mathbf{A}_\alpha + \theta^3\mathbf{A}_{3,\alpha} \\ \mathbf{G}_3 &= \frac{\partial \mathbf{X}}{\partial \theta^3} = \mathbf{A}_3\end{aligned}$$

standard assumption: quadratic terms in θ^3 are discarded

5-parameter model (Reissner-Mindlin)

components of linearized strain tensor

$$
\begin{aligned}
\varepsilon_{11} &= \mathbf{v}_{,1} \cdot \mathbf{A}_1 \\
&+ \theta^3 \left(\mathbf{v}_{,1} \cdot \mathbf{A}_{3,1} + \mathbf{e}_{,1} \times \mathbf{A}_3 \cdot \mathbf{A}_1 + \mathbf{e} \times \mathbf{A}_{3,1} \cdot \mathbf{A}_1 + \mathbf{w}_{s,1} \cdot \mathbf{A}_1 \right) \\
2\varepsilon_{12} &= \mathbf{v}_{,1} \cdot \mathbf{A}_2 + \mathbf{v}_{,2} \cdot \mathbf{A}_1 \\
&+ \theta^3 \left(\mathbf{v}_{,1} \cdot \mathbf{A}_{3,2} + \mathbf{e}_{,1} \times \mathbf{A}_3 \cdot \mathbf{A}_2 + \mathbf{e} \times \mathbf{A}_{3,1} \cdot \mathbf{A}_2 + \mathbf{w}_{s,1} \cdot \mathbf{A}_2 \right) \\
&+ \theta^3 \left(\mathbf{v}_{,2} \cdot \mathbf{A}_{3,1} + \mathbf{e}_{,2} \times \mathbf{A}_3 \cdot \mathbf{A}_1 + \mathbf{e} \times \mathbf{A}_{3,2} \cdot \mathbf{A}_1 + \mathbf{w}_{s,2} \cdot \mathbf{A}_1 \right) \\
\varepsilon_{22} &= \mathbf{v}_{,2} \cdot \mathbf{A}_2 \\
&+ \theta^3 \left(\mathbf{v}_{,2} \cdot \mathbf{A}_{3,2} + \mathbf{e}_{,2} \times \mathbf{A}_3 \cdot \mathbf{A}_2 + \mathbf{e} \times \mathbf{A}_{3,2} \cdot \mathbf{A}_2 + \mathbf{w}_{s,2} \cdot \mathbf{A}_2 \right) \\
2\varepsilon_{13} &= \mathbf{v}_{,1} \cdot \mathbf{A}_3 + \mathbf{e} \times \mathbf{A}_3 \cdot \mathbf{A}_1 + \mathbf{w}_s \cdot \mathbf{A}_1 \\
&+ \theta^3 \left(\mathbf{e}_{,1} \times \mathbf{A}_3 \cdot \mathbf{A}_3 + \mathbf{e} \times \mathbf{A}_{3,1} \cdot \mathbf{A}_3 + \mathbf{w}_{s,1} \cdot \mathbf{A}_3 \right) \\
&+ \theta^3 \left(\mathbf{e} \times \mathbf{A}_3 \cdot \mathbf{A}_{3,1} + \mathbf{w}_s \cdot \mathbf{A}_{3,1} \right) \\
2\varepsilon_{23} &= \mathbf{v}_{,2} \cdot \mathbf{A}_3 + \mathbf{e} \times \mathbf{A}_3 \cdot \mathbf{A}_2 + \mathbf{w}_s \cdot \mathbf{A}_2 \\
&+ \theta^3 \left(\mathbf{e}_{,2} \times \mathbf{A}_3 \cdot \mathbf{A}_3 + \mathbf{e} \times \mathbf{A}_{3,2} \cdot \mathbf{A}_3 + \mathbf{w}_{s,2} \cdot \mathbf{A}_3 \right) \\
&+ \theta^3 \left(\mathbf{e} \times \mathbf{A}_3 \cdot \mathbf{A}_{3,2} + \mathbf{w}_s \cdot \mathbf{A}_{3,2} \right) \\
\varepsilon_{33} &= 0
\end{aligned}
$$

5-parameter model (Reissner-Mindlin)

components of linearized strain tensor

$$
\begin{aligned}
2\varepsilon_{13} &= \mathbf{v}_{,1}\cdot\mathbf{A}_3 + \mathbf{e}\times\mathbf{A}_3\cdot\mathbf{A}_1 + \mathbf{w}_s\cdot\mathbf{A}_1 \\
&+ \theta^3\left(\mathbf{e}_{,1}\times\mathbf{A}_3\cdot\mathbf{A}_3 + \mathbf{e}\times\mathbf{A}_{3,1}\cdot\mathbf{A}_3 + \mathbf{w}_{s,1}\cdot\mathbf{A}_3\right) \\
&+ \theta^3\left(\mathbf{e}\times\mathbf{A}_3\cdot\mathbf{A}_{3,1} + \mathbf{w}_s\cdot\mathbf{A}_{3,1}\right)
\end{aligned}
$$

constant term in θ^3

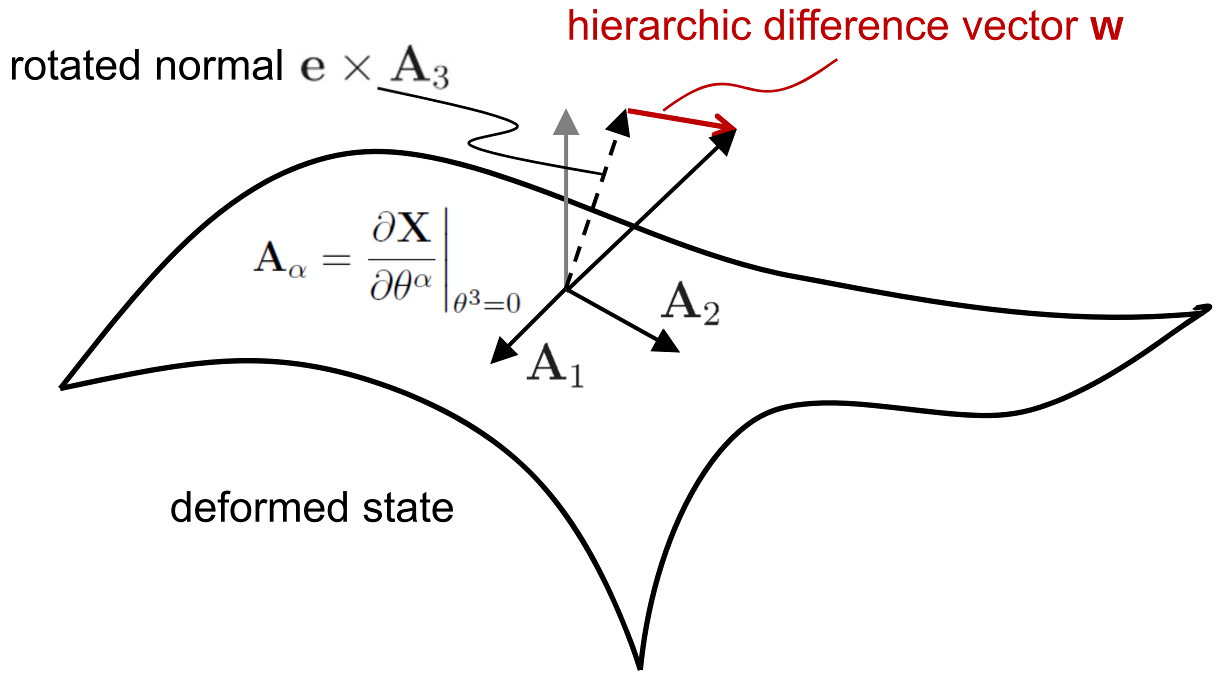
$$
\mathbf{e} = \varphi_1\mathbf{A}_1 + \varphi_2\mathbf{A}_2 \qquad \varphi_1 = \mathbf{v}_{,2}\cdot\mathbf{A}_3 \qquad \varphi_2 = -\mathbf{v}_{,1}\cdot\mathbf{A}_3
$$

$$
\begin{aligned}
\mathbf{e}\times\mathbf{A}_3\cdot\mathbf{A}_1 &= \left(\varphi_1\,\mathbf{A}_1 + \varphi_2\,\mathbf{A}_2\right)\times\mathbf{A}_3\cdot\mathbf{A}_1 \\
&= \varphi_1\,\underbrace{\mathbf{A}_1\times\mathbf{A}_3}_{\mathbf{A}^2}\cdot\mathbf{A}_1 + \varphi_2\,\underbrace{\mathbf{A}_2\times\mathbf{A}_3}_{\mathbf{A}^1}\cdot\mathbf{A}_1 = -\mathbf{v}_{,1}\cdot\mathbf{A}_3
\end{aligned}
$$

similar considerations → linear term in θ^3 vanishes

$$
\Rightarrow\; 2\,\varepsilon_{13} = \mathbf{v}_{,1}\cdot\mathbf{A}_3 - \mathbf{v}_{,1}\cdot\mathbf{A}_3 + \mathbf{w}_s\cdot\mathbf{A}_1 = \mathbf{w}_s\cdot\mathbf{A}_1
$$

7-parameter model (3d-shell)



computation of deformed director

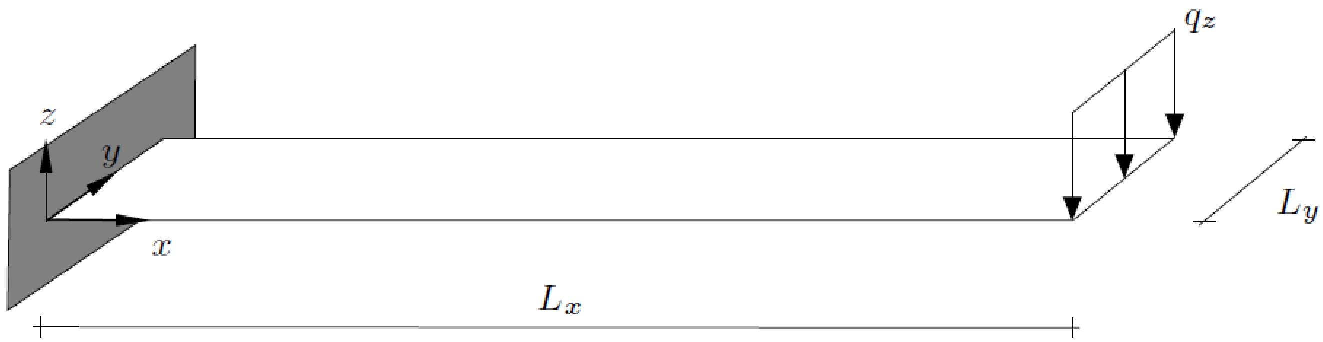
$$\mathbf{a}_3 = \mathbf{A}_3 + \mathbf{e} \times \mathbf{A}_3 + \mathbf{w} \qquad \mathbf{w}_s = \mathbf{w}^\alpha \cdot \mathbf{A}_\alpha$$

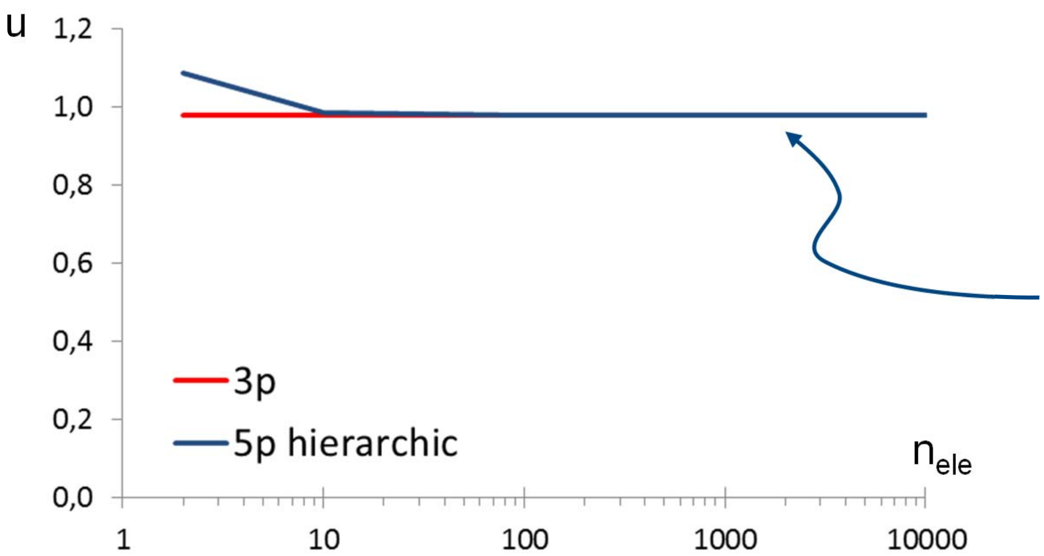
Kirchhoff-Love ($\mathbf{A}_3 + \mathbf{e} \times \mathbf{A}_3$)

+ transverse shear
+ thickness change

3 d.o.f., extensible director
1 additional parameter for linear part of ε_{33} needed

verification: bending of plate strip (cf. beam solution)





normalized displacements vs. number of elements

no shear locking
for hierarchic 5p-elements
(Reissner-Mindlin)

purely displacement based formulation
no transverse shear locking – why?

3-parameter model

$$\mathbf{a}_3 = \mathbf{A}_3 + \mathbf{e} \times \mathbf{A}_3$$

no transverse shear strains
→ no transverse shear locking

5-parameter model

$$\mathbf{a}_3 = \mathbf{A}_3 + \mathbf{e} \times \mathbf{A}_3 + \mathbf{w}_s$$

no transverse shear:

$\mathbf{w}_s = 0$ easily satisfied!

7-parameter model

$$\mathbf{a}_3 = \mathbf{A}_3 + \mathbf{e} \times \mathbf{A}_3 + \mathbf{w}$$

no transverse shear, no thickness change:

$\mathbf{w} = 0$ easily satisfied!

explanation for model problem: Timoshenko beam

shear angle must vanish in the thin limit: $w' = -\varphi$

standard formulation with rotations

$$\gamma = w' + \varphi$$

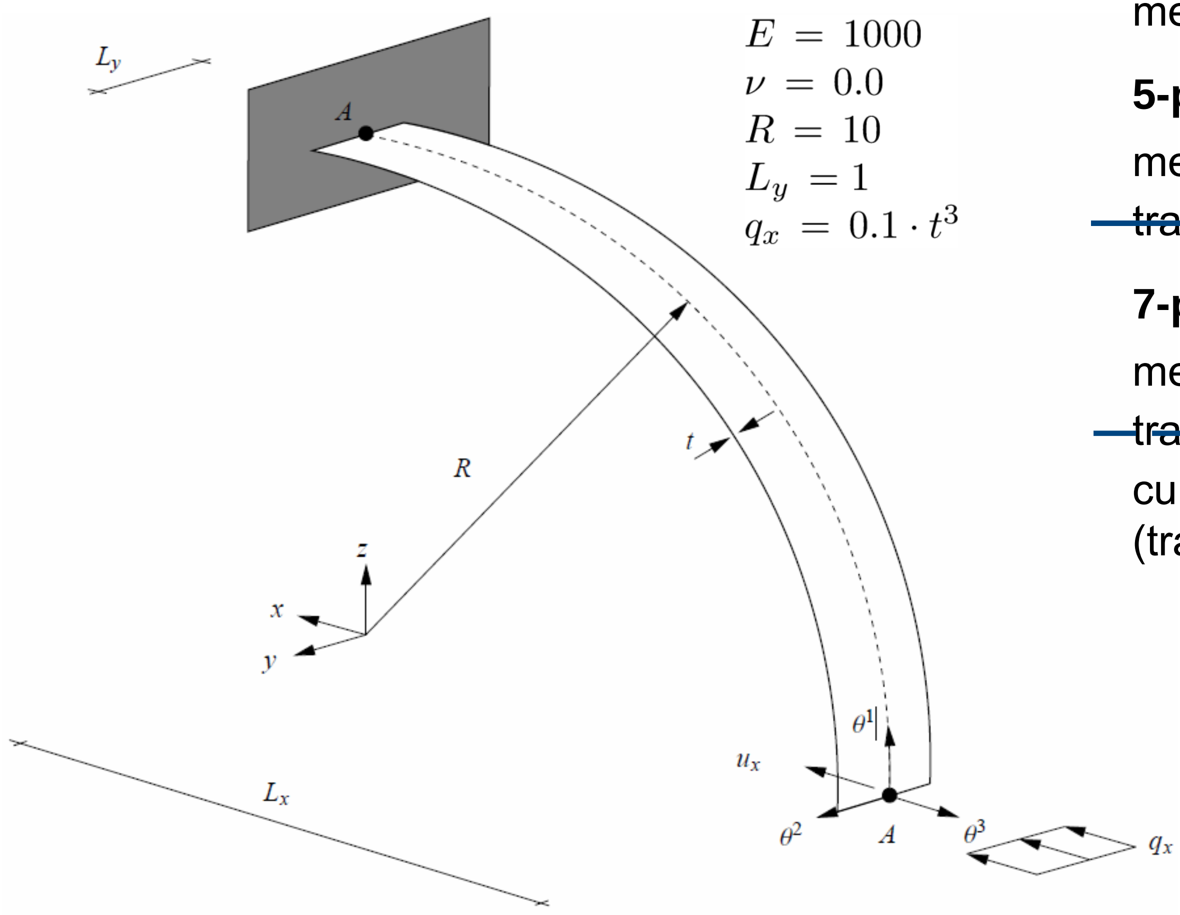
unbalanced shape functions cause locking

formulation with hierarchic rotations

$$\gamma = w' + (-w' + \bar{\varphi}) = \bar{\varphi}$$

rotated normal

bending of cylindrical shell strip



$E = 1000$
$\nu = 0.0$
$R = 10$
$L_y = 1$
$q_x = 0.1 \cdot t^3$

expected locking problems

3-parameter model

membrane locking

5-parameter model

membrane locking

~~transverse shear locking~~

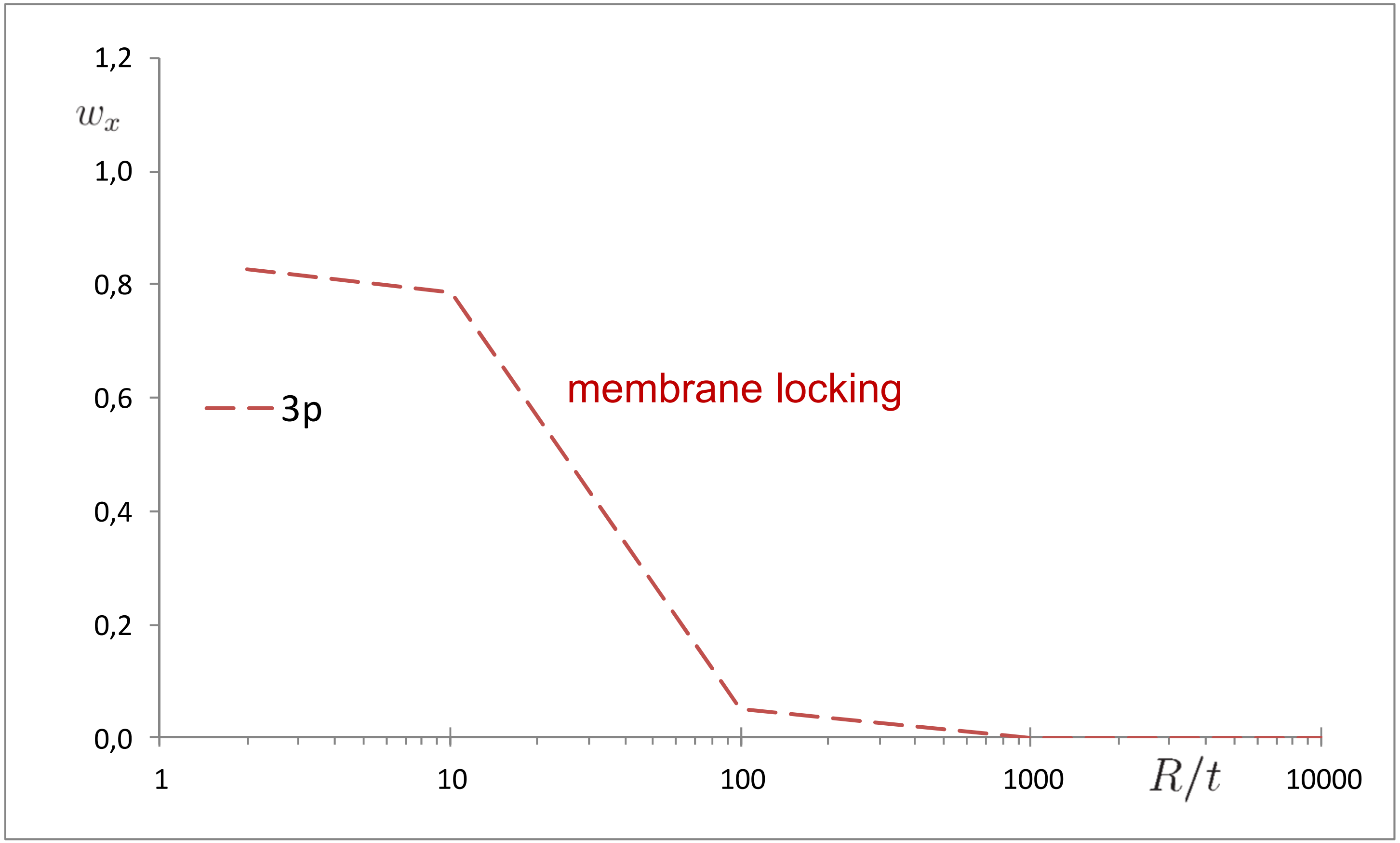
7-parameter model

membrane locking

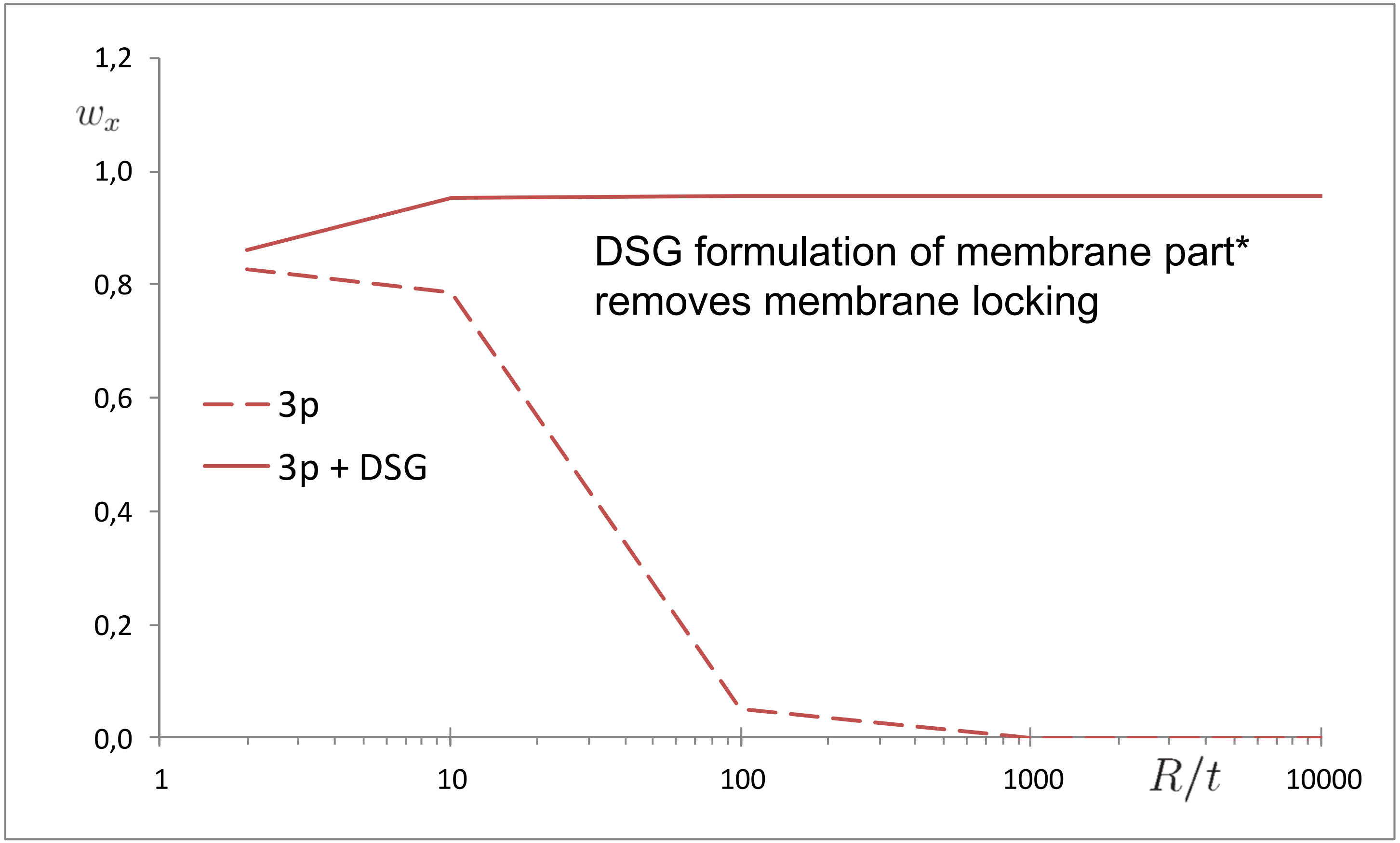
~~transverse shear locking~~

curvature thickness locking
(trapezoidal locking)

bending of cylindrical shell strip

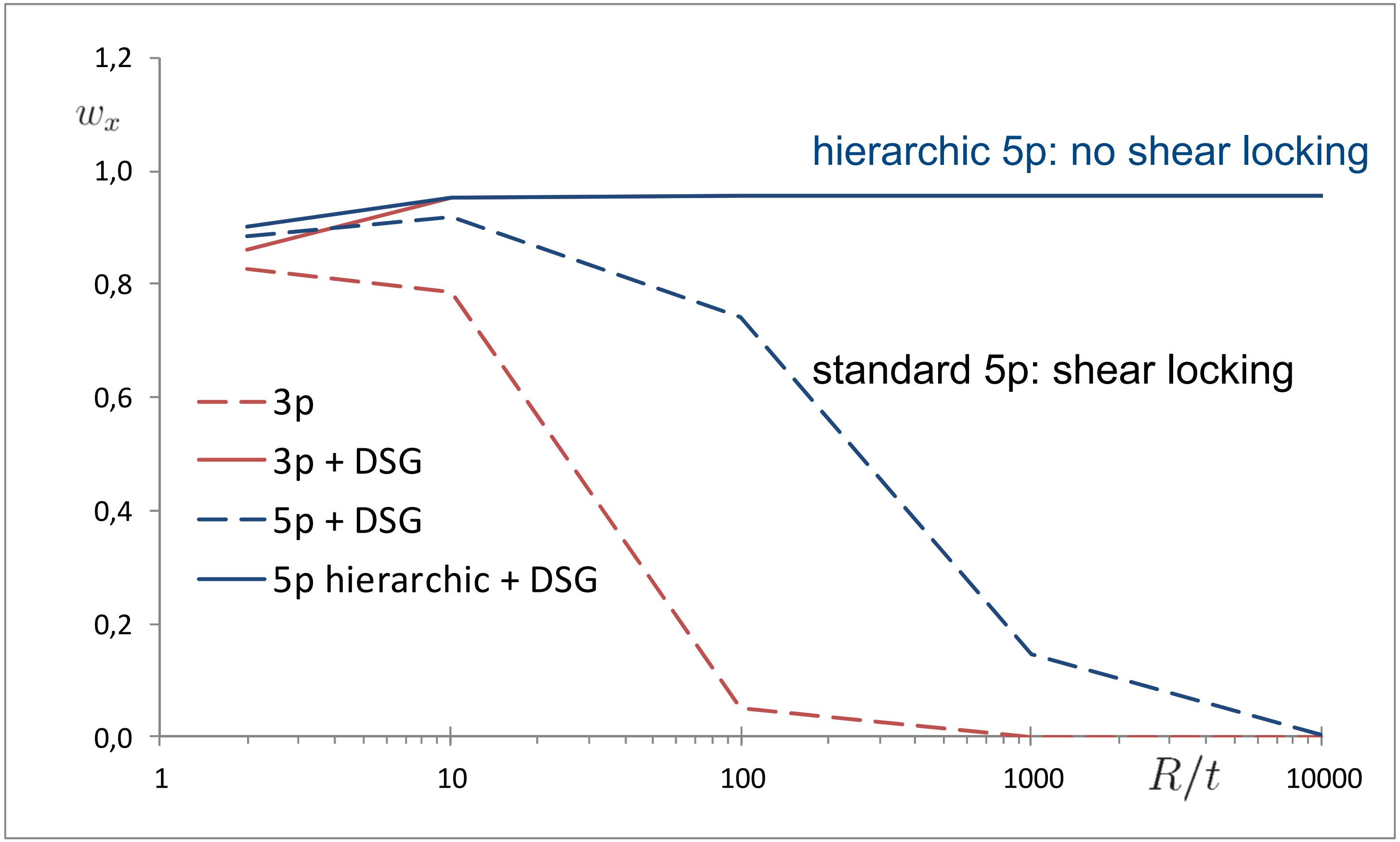


bending of cylindrical shell strip

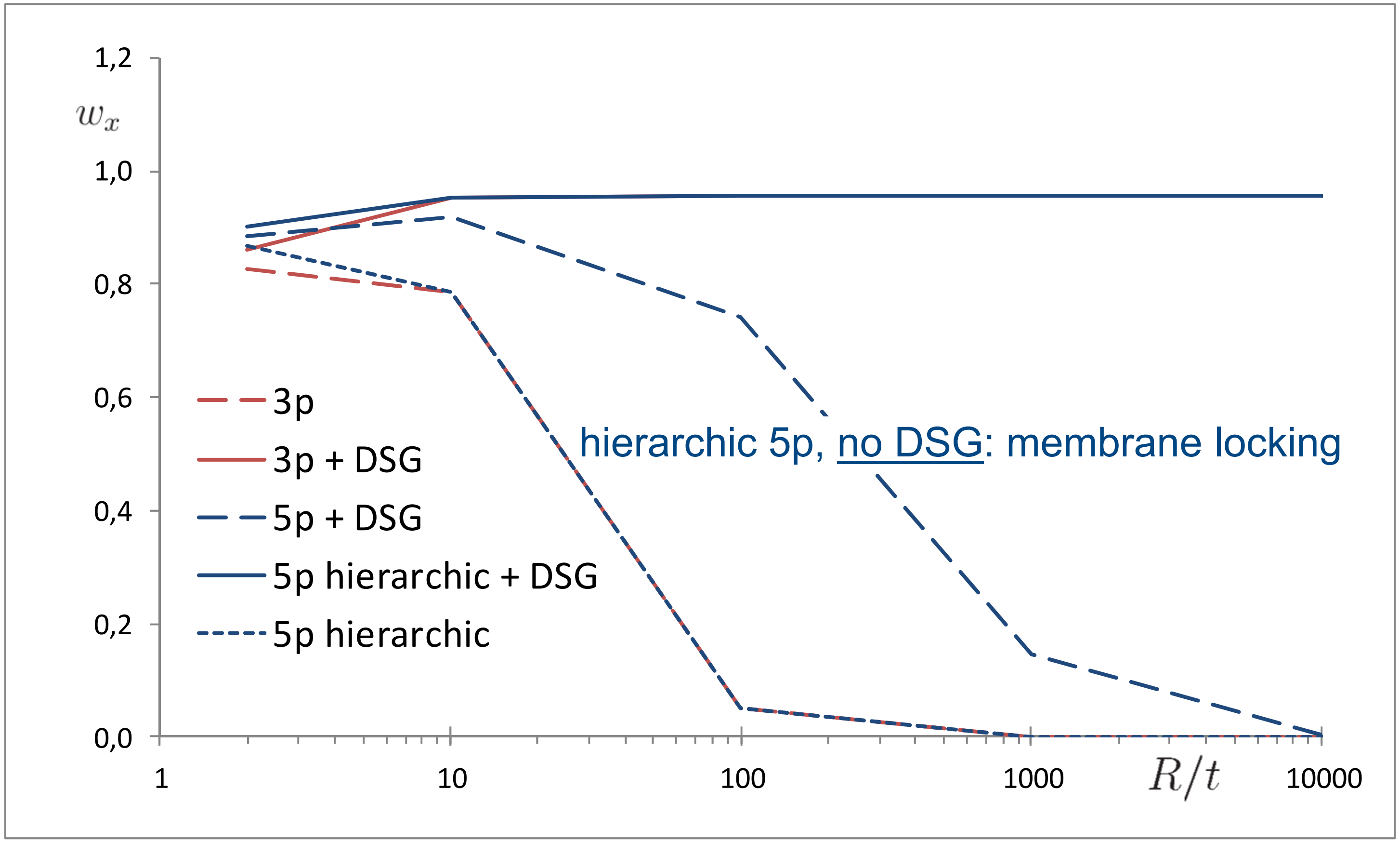


*KOSCHNICK, BISCHOFF, CAMPRUBI, BLETZINGER [2005]
ECHTER, BISCHOFF [2010]

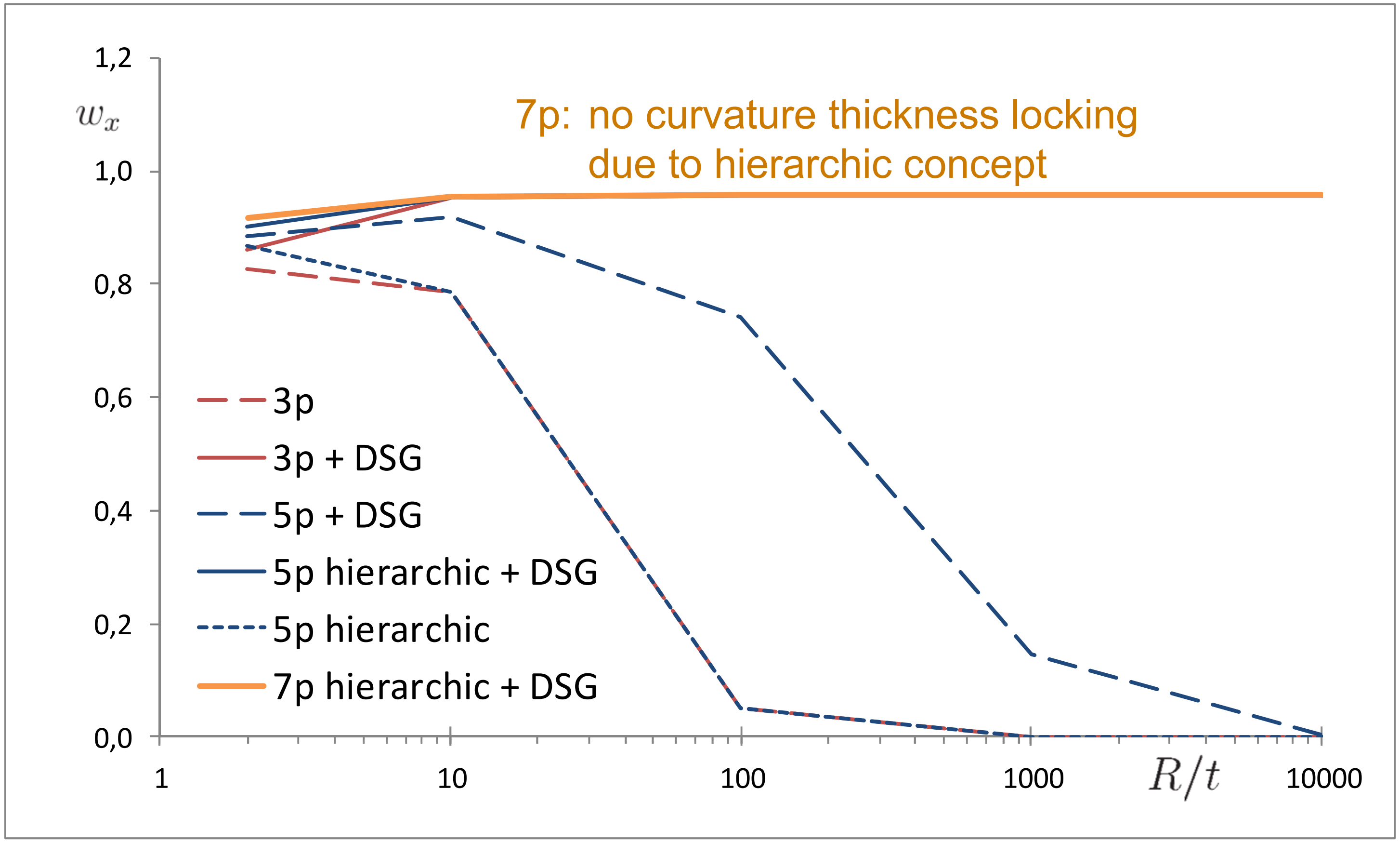
bending of cylindrical shell strip



bending of cylindrical shell strip



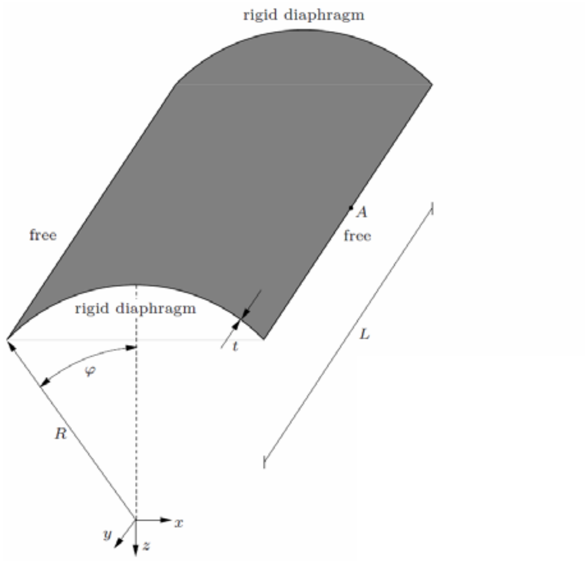
bending of cylindrical shell strip



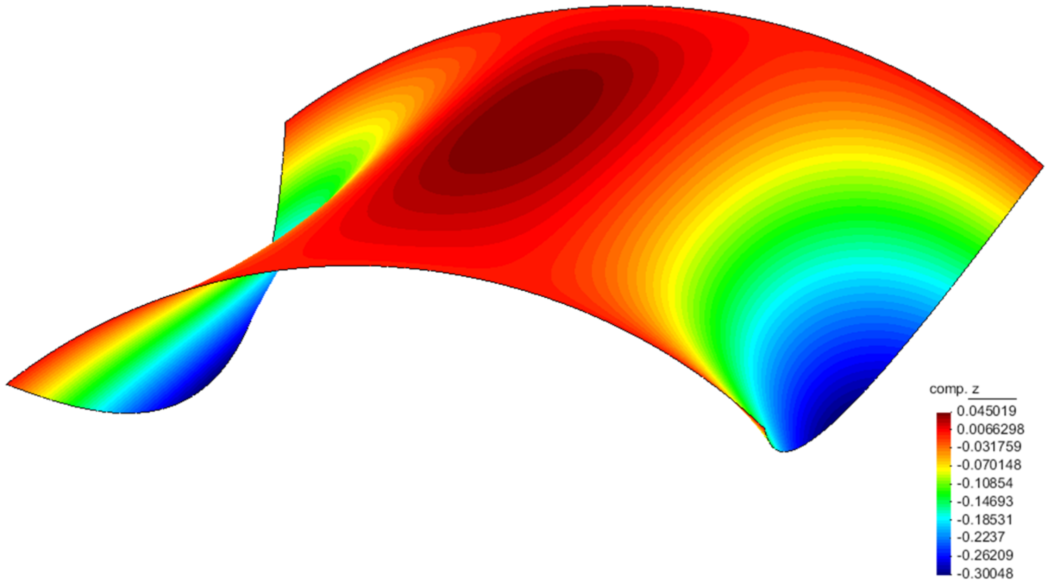
Performance of Isogeometric Shell Elements

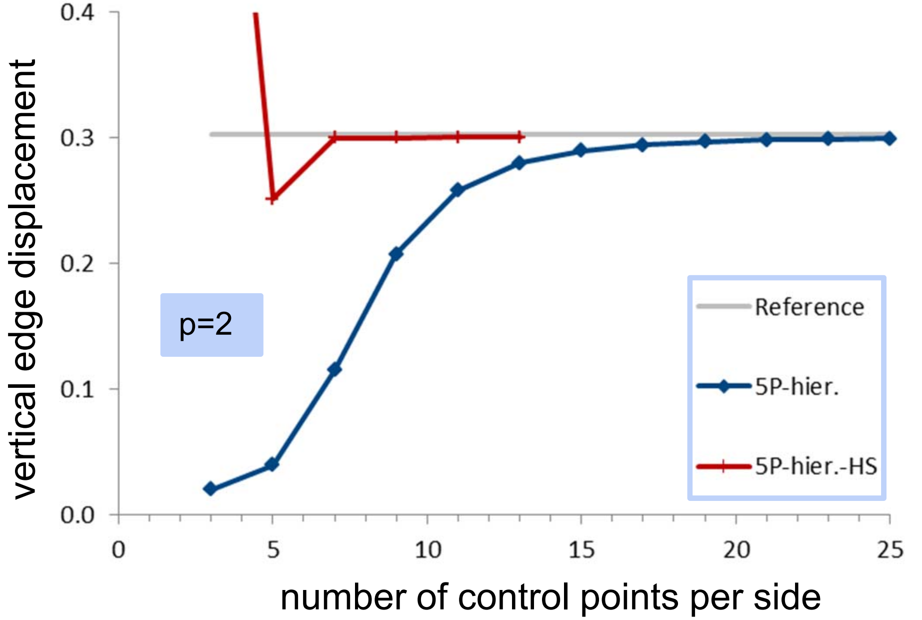
Scordelis-Lo roof

avoiding membrane locking
via hybrid stress method (HS)
as alternative to DSG



$$E = 4.32 \cdot 10^8$$
$$\nu = 0.0$$
$$L = 50$$
$$R = 25$$
$$t = 0.25$$
$$\varphi = 40^\circ$$





a hierarchic family of isogeometric shell finite elements

C^1-continuous Reissner-Mindlin and 3d-shells

unique “nodal” director has several remarkable benefits!

finite element performance, locking

hierarchic concept naturally removes locking

DSG formulation removes membrane locking
(alternative: mixed method, not shown here)

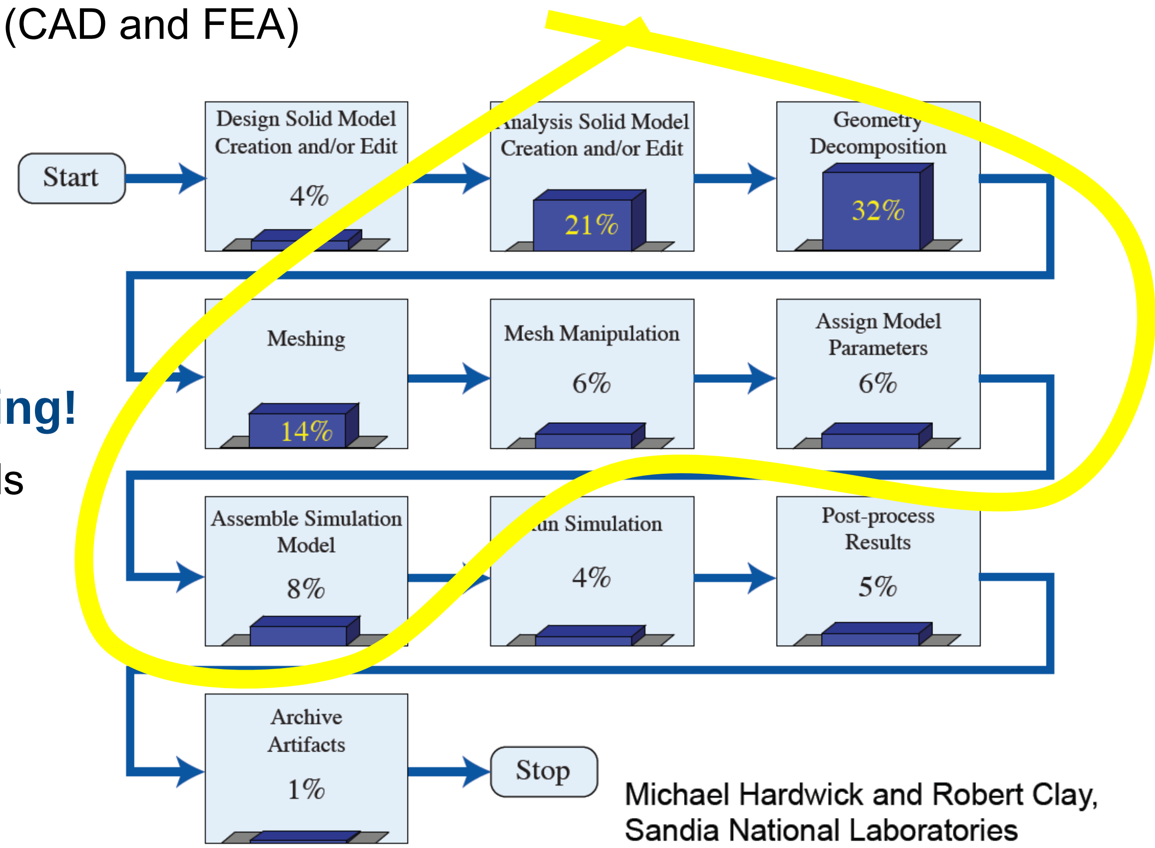
way forward

geometrically non-linear, several patches, boundary conditions…

hierarchic concept to remove membrane locking?

automatic transfer from CAD to FEA?

from design model to analysis results (CAD and FEA)



Michael Hardwick and Robert Clay, Sandia National Laboratories

not only "meshing", but modeling!

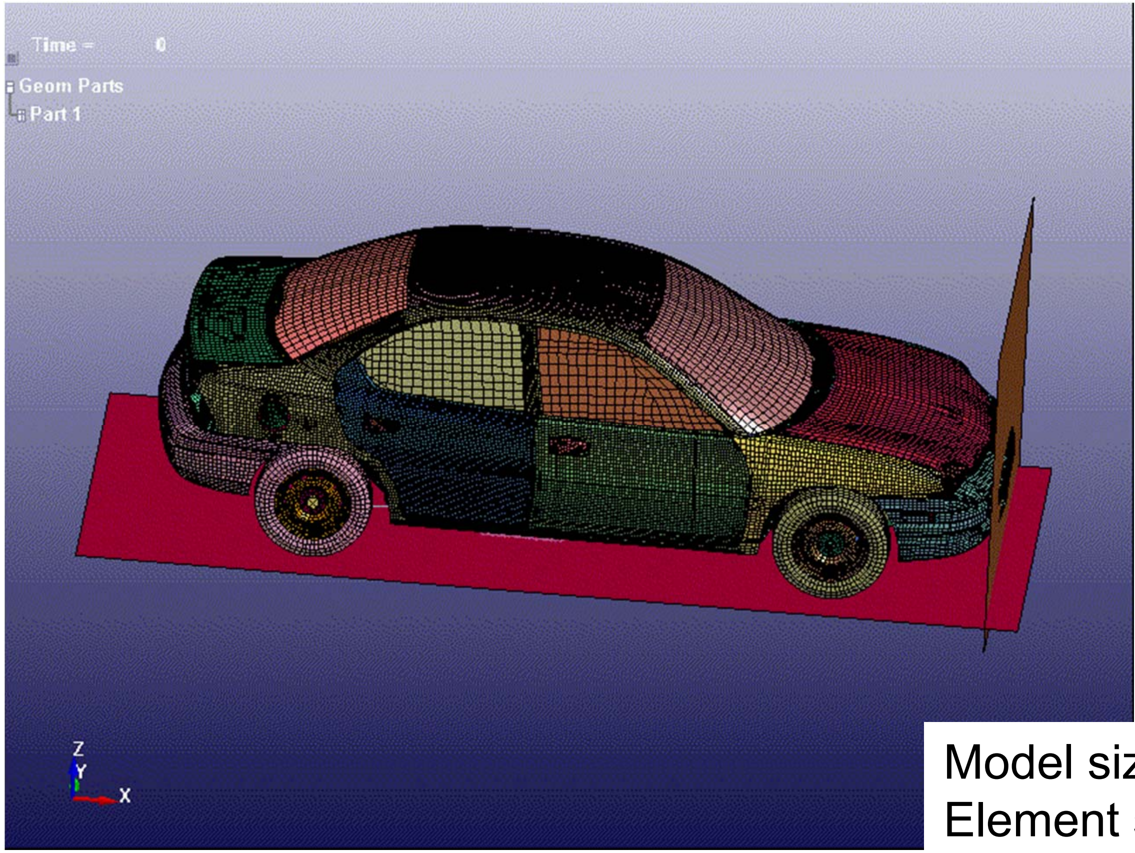
- removing irrelevant geometric details
- forces and boundary conditions
- connections, d.o.f. coupling
- assembling structural models of different dimensionality

HUGHES, COTTRELL, BAZILEVS. ISOGEOMETRIC ANALYSIS: CAD, FINITE ELEMENTS, NURBS, EXACT GEOMETRY AND MESH REFINEMENT. COMPUTER METHODS IN APPLIED MECHANICS AND ENGINEERING 194, 4135-4195, 2005.

Baustatik und Baudynamik
Universität Stuttgart

full car crash, LS-DYNA

relatively coarse discretization, much finer models are used today



Model size:	1 Mio DOF
Element size:	5 mm (mainly shells)
Time-step:	1 µs
Duration of crash:	150 ms
Computation time:	12 hours

full car crash, LS-DYNA

analysis of computational expense (first 40 ms)
one node, four cores

efficient finite elements
adaptive mesh refinement
sub-cycling
reduced order modeling

T i m i n g   i n f o r m a t i o n

	CPU(seconds)	%CPU	Clock(seconds)	%Clock
Initialization	2.4000E+01	0.10	2.3306E+01	0.09
Element processing ...	1.7457E+04	70.75	1.7468E+04	70.80
Binary databases	6.2000E+01	0.25	6.9039E+01	0.28
ASCII database	2.4000E+01	0.10	1.8748E+01	0.08
Contact algorithm	7.0830E+03	28.71	7.0711E+03	28.66
Rigid bodies	2.4000E+01	0.10	2.3625E+01	0.10
T o t a l s	2.4674E+04	100.00	2.4674E+04	100.00

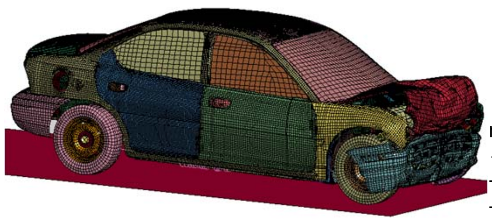
```
Problem time       =    4.0001E+01
Problem cycle      =     63493
Total CPU time     =     24674 seconds (   6 hours 51 minutes   )
```

increase size of critical time step
via mass scaling

aim: increasing efficiency while retaining accuracy

explicit dynamics
reduce CPU time

- efficient elements
- adaptive mesh refinement
- subcycling
- reduced order modeling
- selective mass scaling



o n

	nds)	%CPU	Clock(seconds)	%Clock
	+01	0.10	2.3306E+01	0.09
	+04	70.75	1.7468E+04	70.80
Binary databases	6.2000E+01	0.25	6.9039E+01	0.28
ASCII database	2.4000E+01	0.10	1.8748E+01	0.08
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Problem time      =   4.0001E+01
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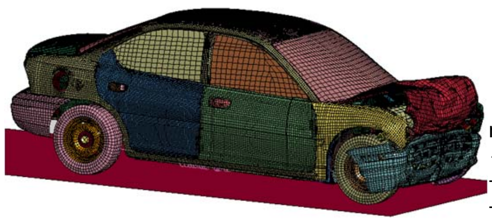
```

aim: increasing efficiency while retaining accuracy

explicit dynamics
reduce CPU time

- efficient elements
- adaptive mesh refinement
- subcycling
- reduced order modeling
- selective mass scaling



o n

	nds)	%CPU	Clock(seconds)	%Clock
	+01	0.10	2.3306E+01	0.09
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Binary databases	6.2000E+01	0.25	6.9039E+01	0.28
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Rigid bodies	2.4000E+01	0.10	2.3625E+01	0.10
T o t a l s	2.4674E+04	100.00	2.4674E+04	100.00

Problem time = 4.0001E+01
Problem cycle = 63493
Total CPU time = 24674 seconds (6 hours 51 minutes)

how does mass scaling work?

$$\omega_{\max}^{-1} \sim \Delta t_{\text{crit}} \simeq \frac{l_{\min}}{c}$$

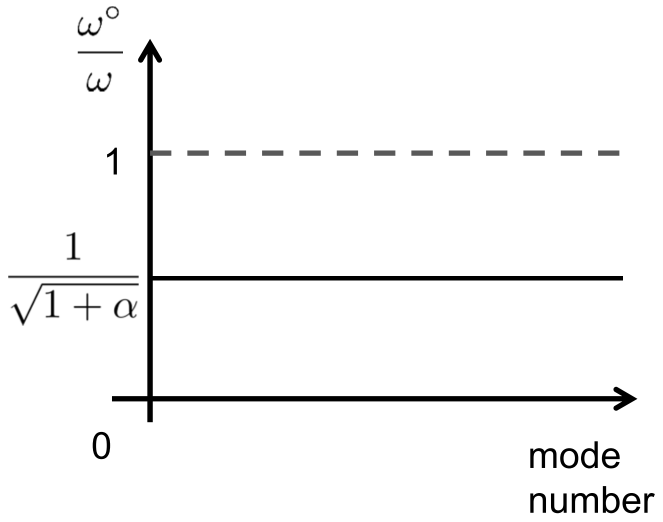
larger maximum frequency ⇔ smaller time step

conventional mass scaling, since 1970s

adding artificial mass in diagonal terms of mass matrix

$$\mathbf{m}_{\mathrm{e}} = \frac{\rho A l_{\mathrm{e}}}{2} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\mathbf{M} = \bigcup_{\mathrm{e}} (1+\alpha)\mathbf{m}_{\mathrm{e}}$$



- preserving diagonal structure of mass matrix
- increasing element inertia
- usually applied to a small number of selected (stiff) elements

how does mass scaling work?

$$\omega_{\max}^{-1} \sim \Delta t_{\text{crit}} \simeq \frac{l_{\min}}{c}$$

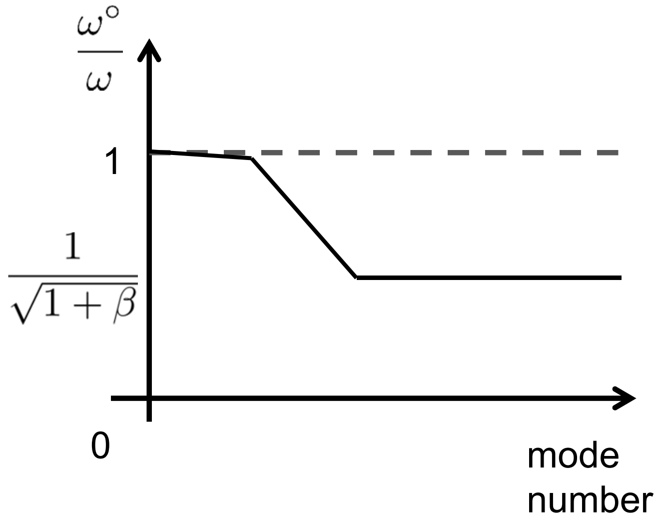
larger maximum frequency ⇔ smaller time step

selective mass scaling (SMS), since 2004, e.g. in RADIOSS, LS-DYNA

adding artificial mass but **preserving translational inertia**

$$\boldsymbol{\lambda}_e^\circ = \beta \frac{\rho A l_e}{2} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$\mathbf{M}^\circ = \bigcup_e (\mathbf{m}_e + \boldsymbol{\lambda}_e^\circ)$$



- only selected modes are influenced
- off-diagonal terms in mass matrix
- solution of $\mathbf{a} = \mathbf{M}^{-1}\mathbf{f}$ needed in each time step

history

2004	SMS in explicit FEA for thin-walled structures modeled with solids[1]
2005	general method[2]
2006	PCG with Jacobi preconditioner for acceleration ($a=M^{-1}f$) [3]
2006	LS-DYNA[6]
2009	RADIOSS
2012	Impetus AFEA
2013	variational formulation for SMS[4], templates for mass matrix[7]

range of applications

metal forming (initially)
car crash (frontal, side, rollover)
drop tests
metal cutting
human models
solid shells[5], distorted elements

[1] Olovsson et al. (2004). Selective mass scaling for thin walled structures modeled with tri-linear solid elements. Computational Mechanics, 34

[2] Olovsson et al. (2005) Selective mass scaling for explicit finite element analyses, IJNME 63

[3] Olovsson & Simonsson (2006). Iterative solution technique in selective mass scaling, Comm. num. meth. engrg., 22

[4] Tkachuk & Bischoff (2013). Variational methods for selective mass scaling. Comput Mech 52

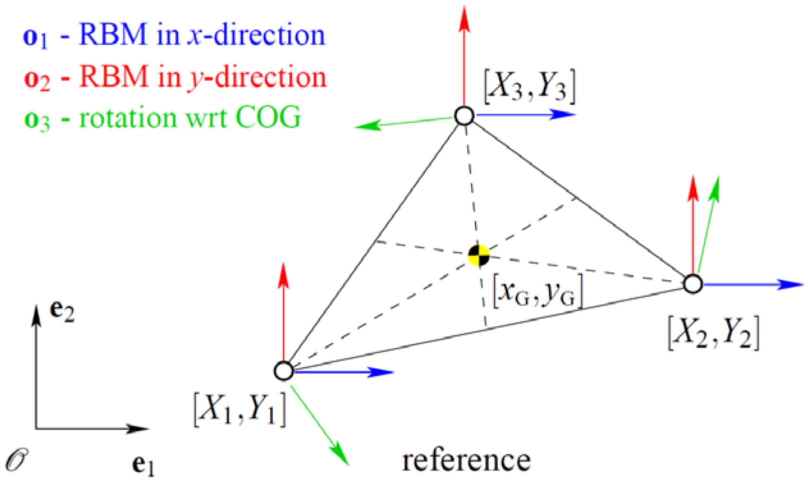
[5] Cocchetti et al. (2012). Selective mass scaling and critical time-step estimate for explicit dynamics analyses with solid-shell elements. Computers & Structures 127

[6] Borrvall (2012). U.S. Patent No. 20,120,323,536. Washington, DC: U.S. Patent and Trademark Office.

[7] Felippa et al. (2013). Mass Matrix Templates: General Descriptions and 1D Examples. Arch. Comput. Methods. Eng.

example: 3-node plane stress element

lumped mass matrix (diagonal)



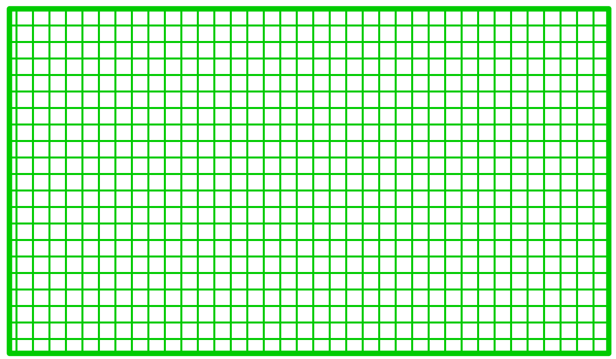
$$\mathbf{m}_e = \frac{m}{3}\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{o}_1 = 1/\sqrt{3}\begin{bmatrix} 1 & 0 & 1 & 0 & 1 & 0 \end{bmatrix}^{\mathrm{T}}$$

$$\mathbf{o}_2 = 1/\sqrt{3}\begin{bmatrix} 0 & 1 & 0 & 1 & 0 & 1 \end{bmatrix}^{\mathrm{T}}$$

$$\lambda_e^\circ = \frac{\Delta m_e}{n-1}\left(\mathbf{I} - \sum_i \mathbf{o}_i \mathbf{o}_i^T\right)$$

$$\boldsymbol{\lambda}_e^\circ = \frac{\Delta m_e}{6}$$



- existing methods may violate consistency
- difficult to prove convergence
- potentially undesired effect on results
- difficult to generalize (e.g. for high-order or isogeometric finite elements)

OLOVSSON ET AL. (2005) SELECTIVE MASS SCALING FOR EXPLICIT FINITE ELEMENT ANALYSES, IJNME 63

issues: existing methods…

- …may violate consistency
- …are difficult to generalize
- …may effect computational results in an undesired way

variational formulation is desirable, to…

- …design consistent methods
- …create a general framework (independent of element type, problem type)
- …understand underlying nature of the methods

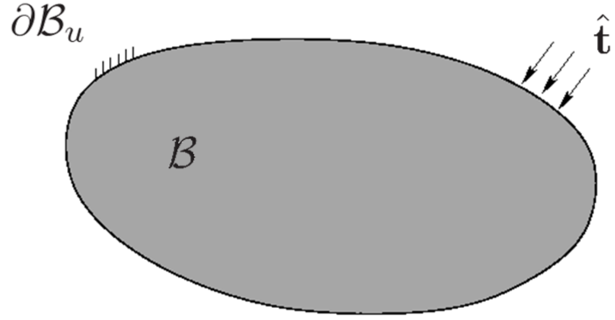
proposed approach relies on …

- …multi-field variational principles
- …independent fields are linked in weak sense or via penalty methods
- …different ansatz spaces for u, v, p
- …additional considerations for selective mass scaling and singular mass matrices

Penalized Hamilton's Principle

strong form vs. weak form

$$
\begin{cases}
\rho\ddot{\mathbf{u}} = \mathbf{L}^{*}\boldsymbol{\sigma}_{\mathrm{lin}}(\mathbf{u}) + \hat{\mathbf{b}} & \text{in } \mathcal{I}\times\mathcal{B}_0 \\
\boldsymbol{\sigma}_{\mathrm{lin}} = \mathbf{D}\boldsymbol{\varepsilon} & \text{in } \mathcal{I}\times\mathcal{B}_0 \\
\boldsymbol{\varepsilon} = \mathbf{L}\mathbf{u} & \text{in } \mathcal{I}\times\mathcal{B}_0 \\
\mathbf{u} = \mathbf{0} & \text{in } \mathcal{I}\times\partial\mathcal{B}_u \\
\boldsymbol{\sigma}_{\mathrm{lin}}\mathbf{n} = \hat{\mathbf{t}} & \text{in } \mathcal{I}\times\partial\mathcal{B}_{t,0} \\
\mathbf{u}(0,.) = \mathbf{u}_0 & \text{in } \mathcal{B}_0 \\
\dot{\mathbf{u}}(0,.) = \mathbf{v}_0 & \text{in } \mathcal{B}_0.
\end{cases}
$$



$$\mathcal{I} = [0, t_{\mathrm{end}}]$$

Hamilton's principle

$$H(\mathbf{u}) = \int_{\mathcal{I}} \left(T - \Pi^{\mathrm{int}} + \Pi^{\mathrm{ext}}\right)\, \mathrm{d}t \to \mathrm{stat}$$

with

$$T(\dot{\mathbf{u}}) = \frac{1}{2}\int_{\mathcal{B}_0} \rho\dot{\mathbf{u}}\cdot\dot{\mathbf{u}}\,\mathrm{d}V$$

$$\Pi^{\mathrm{int}}(\mathbf{u}) = \frac{1}{2}\int_{\mathcal{B}_0} \boldsymbol{\varepsilon}(\mathbf{u})\cdot\mathbf{D}\,\boldsymbol{\varepsilon}(\mathbf{u})\,\mathrm{d}V$$

$$\Pi^{\mathrm{ext}}(\mathbf{u}) = \int_{\mathcal{B}_0} \hat{\mathbf{b}}\cdot\mathbf{u}\,\mathrm{d}V + \int_{\partial\mathcal{B}_{t,0}} \hat{\mathbf{t}}\cdot\mathbf{u}\,\mathrm{d}A$$

TKACHUK, A. VARIATIONAL METHODS FOR CONSISTENT SINGULAR AND SCALED MASS MATRICES.
PHD DISSERTATION, INSTITUT FÜR BAUSTATIK UND BAUDYNAMIK, UNIVERSITÄT STUTTGART, 2013.

following Carlos Felippas's idea
from finite element technology

penalty factors

$$T^\circ = \underbrace{\frac{1}{2}\int_{\mathcal{B}} \rho\, \dot{\mathbf{u}}^2 \,\mathrm{d}V}_{\text{kinetic energy}} + \int_{\mathcal{B}} \frac{C_1}{2\rho}(\underbrace{\rho\dot{\mathbf{u}} - \mathbf{p}}\,)^2 + \frac{C_2}{2\rho}(\underbrace{\rho\mathbf{v} - \mathbf{p}}\,)^2 + \frac{C_3\rho}{2}(\underbrace{\mathbf{v} - \dot{\mathbf{u}}}\,)^2 \,\mathrm{d}V$$

kinetic energy

algebraic conditions linking the fields

penalized Hamilton's principle

$$H^\circ(\mathbf{u}, \mathbf{v}, \mathbf{p}, C_1, C_2, C_3) = \int_{\mathcal{I}} \left(T^\circ - \Pi^{\mathrm{int}} + \Pi^{\mathrm{ext}}\right) \,\mathrm{d}t \rightarrow \mathrm{stat}$$

template for kinetic energy

$$T^\circ = \frac{1}{2}\int_{\mathcal{B}} \begin{bmatrix} \rho\dot{\mathbf{u}} \\ \rho\mathbf{v} \\ \mathbf{p} \end{bmatrix}^T \underbrace{\begin{bmatrix} (1 + C_1 + C_3)\mathbf{I} & -C_3\mathbf{I} & -C_1\mathbf{I} \\ -C_3\mathbf{I} & (C_2 + C_3)\mathbf{I} & -C_2\mathbf{I} \\ -C_1\mathbf{I} & -C_2\mathbf{I} & (C_1 + C_2)\mathbf{I} \end{bmatrix}} \underbrace{\begin{bmatrix} \dot{\mathbf{u}} \\ \mathbf{v} \\ \frac{\mathbf{p}}{\rho} \end{bmatrix}} \,\mathrm{d}V$$

functional generating matrix

generalized field vector

FELIPPA (1994). A SURVEY OF PARAMETRIZED VARIATIONAL PRINCIPLES AND APPLICATIONS TO COMPUTATIONAL MECHANICS. COMPUTER METHODS IN APPLIED MECHANICS AND ENGINEERING 113

FELIPPA ET AL. (2013). MASS MATRIX TEMPLATES: GENERAL DESCRIPTIONS AND 1D EXAMPLES. ARCH. COMPUT. METHODS. ENG.

TKACHUK & BISCHOFF (2013). VARIATIONAL METHODS FOR SELECTIVE MASS SCALING. COMPUT MECH 52

specification of parameters $C_3 = 0, \quad C_1 = -C_2$

three-field formulation

$$\mathbf{u}^{\mathrm{h}} = \mathbf{N}\mathbf{U} \qquad \mathbf{v}^{\mathrm{h}} = \boldsymbol{\Psi}\mathbf{V} \qquad \mathbf{p}^{\mathrm{h}} = \boldsymbol{\chi}\mathbf{P}$$

semi-discrete equation of motion

$$\begin{cases} (1+C_1)\mathbf{M}\ddot{\mathbf{U}} - C_1\mathbf{A}\dot{\mathbf{P}} + \mathbf{K}\mathbf{U} & = \mathbf{F}^{\mathrm{ext}} \\ \mathbf{C}\mathbf{V} & = \mathbf{B}\mathbf{P} \\ \mathbf{B}^{\mathrm{T}}\mathbf{V} & = \mathbf{A}^{\mathrm{T}}\dot{\mathbf{U}}. \end{cases}$$

with the following matrices

$$\mathbf{M} = \int_{\mathscr{B}_0} \rho_0 \mathbf{N}^{\mathrm{T}}\mathbf{N}\,\mathrm{d}V \quad \mathbf{A} = \int_{\mathscr{B}} \mathbf{N}^{\mathrm{T}}\boldsymbol{\chi}\,\mathrm{d}V \quad \mathbf{B} = \int_{\mathscr{B}} \boldsymbol{\Psi}^{\mathrm{T}}\boldsymbol{\chi}\,\mathrm{d}V \quad \mathbf{C} = \int_{\mathscr{B}_0} \rho_0 \boldsymbol{\Psi}^{\mathrm{T}}\boldsymbol{\Psi}\,\mathrm{d}V$$

selectively scaled mass matrix as result of elimination

$$\mathbf{M}^{\circ}\ddot{\mathbf{U}} + \mathbf{K}\mathbf{U} = \mathbf{F}^{\mathrm{ext}}$$

$$\mathbf{M}^{\circ} = \mathbf{M} + \boldsymbol{\lambda}^{\circ}$$

$$\boldsymbol{\lambda}^{\circ} = C_1\left(\mathbf{M} - \mathbf{A}(\mathbf{B}^{\mathrm{T}}\mathbf{C}^{-1}\mathbf{B})^{-1}\mathbf{A}^{\mathrm{T}}\right)$$

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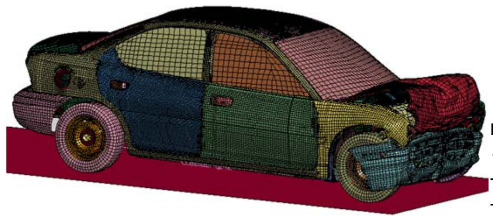
variational selective mass scaling

expected benefits
speed-up due to larger critical time step
consistent, general framework

potential limitations
extra expense due to non-diagonal mass
loss of accuracy



```
                                  o n
                                 nds)    %CPU   Clock(seconds) %Clock
                                 ----------------------------------
                                 +01     0.10     2.3306E+01    0.09
                                 +04    70.75     1.7468E+04   70.80
 Binary databases ..... 6.2000E+01    0.25     6.9039E+01    0.28
 ASCII database ....... 2.4000E+01    0.10     1.8748E+01    0.08
 Contact algorithm .... 7.0830E+03   28.71     7.0711E+03   28.66
 Rigid bodies ......... 2.4000E+01    0.10     2.3625E+01    0.10
---------------------------------------------------------------
 T o t a l s            2.4674E+04  100.00     2.4674E+04  100.00

Problem time       =    4.0001E+01
Problem cycle      =     63493
Total CPU time     =     24674 seconds (   6 hours 51 minutes  )
```

estimate of speed-up

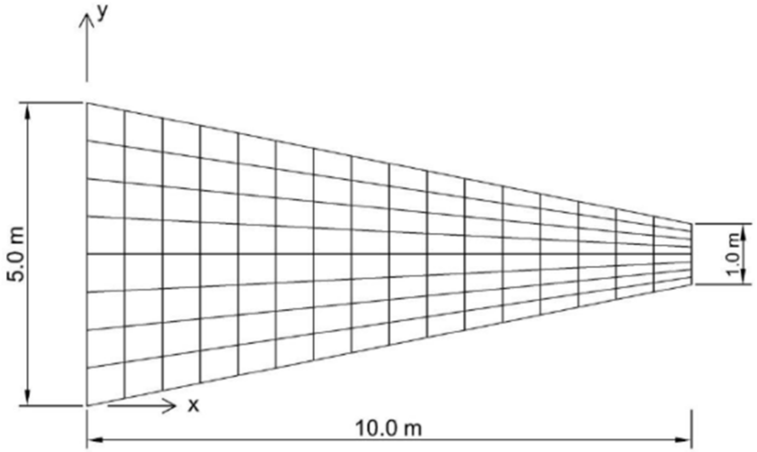
1. Initialize $t = t_0$, $\mathbf{U} = \mathbf{U}_0$, $\dot{\mathbf{U}} = \dot{\mathbf{U}}_0$
2. Compute LMM $\mathbf{M}$ or SMS $\mathbf{M}^{\circ}$ and preconditioner for mass matrix $\boldsymbol{P}$
3. Get global force vector $\mathbf{F}_n = \mathbf{F}_n^{\text{ext}} - \mathbf{F}_n^{\text{int}} - \mathbf{F}_n^{\text{vbc}}$
4. Compute acceleration $\ddot{\mathbf{U}}_n = \mathbf{M}^{-1}\mathbf{F}_n$
5. Time update $t_{n+1} = t_n + \triangle t$
6. Partial update of velocity $\dot{\mathbf{U}}_{n+1/2} = \dot{\mathbf{U}}_n + \frac{\triangle t}{2}\ddot{\mathbf{U}}_n$
7. Enforce velocity b.c. $\dot{\mathbf{U}}_{n+1/2} = \hat{\dot{\mathbf{U}}}_{n+1/2}$
8. Update nodal displacements $\mathbf{U}_{n+1} = \mathbf{U}_n + \triangle t \dot{\mathbf{U}}_{n+1/2}$
9. Get global force vector $\mathbf{F}_{n+1} = \mathbf{F}_{n+1}^{\text{ext}} - \mathbf{F}_{n+1}^{\text{int}} - \mathbf{F}_{n+1}^{\text{vbc}}$
10. Compute acceleration $\ddot{\mathbf{U}}_{n+1} = \mathbf{M}^{-1}\mathbf{F}_{n+1}$
11. Partial update of velocity $\dot{\mathbf{U}}_{n+1} = \dot{\mathbf{U}}_{n+1/2} + \frac{\triangle t}{2}\ddot{\mathbf{U}}_{n+1}$
12. Update time-step counter to $n+1$
13. Output

14 If $t_{n+1} < t_{\text{end}}$ go to 5.

$$\textbf{Speed-up} = \frac{\triangle t^{\text{SMS}}}{\triangle t^{\text{LMM}}(1 + t_{\text{solver}}/t_{\text{element}})}$$

NAFEMS FV32 benchmark

computation of eigenvalues

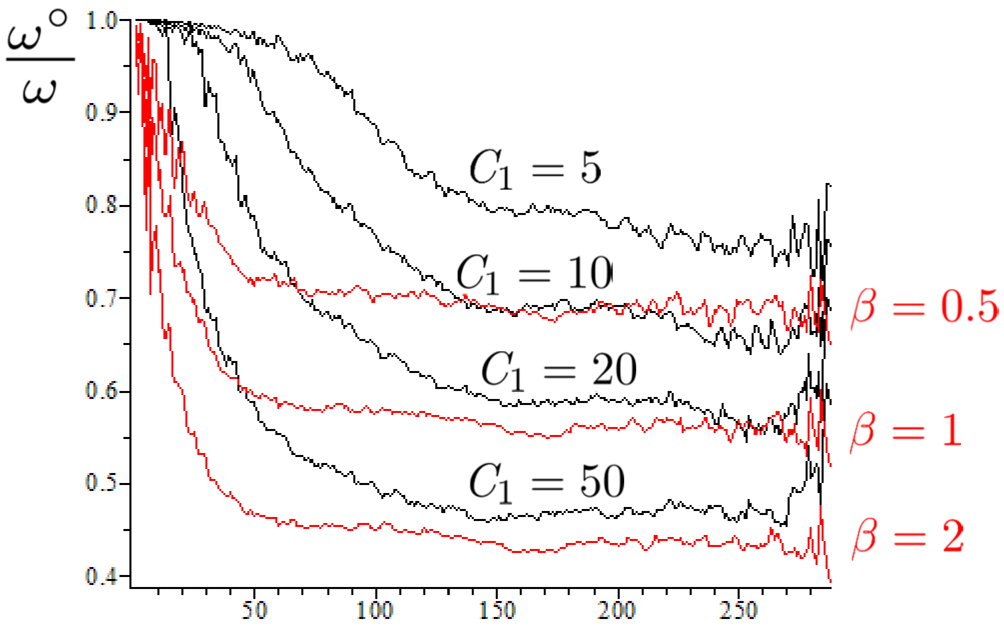


E = 200 GPa
ν = 0.3

ρ = 8000 kg/m^3
t = 0.05 m

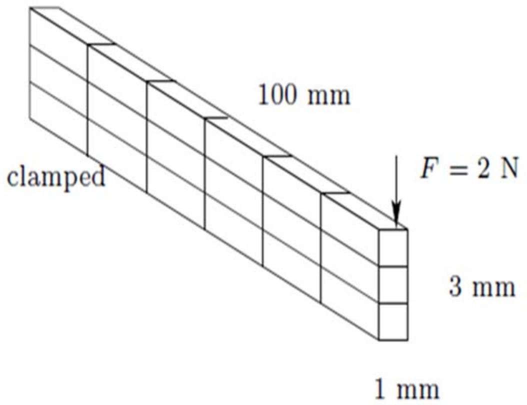
penalized Hamilton's principle with various different values for C_1

Olovsson's mass scaling with various different values for β



Olovsson et al. (2005) Selective mass scaling for explicit finite element analyses, IJNME 63

cantilever beam



system data:

$E = 207$ GPa

$\nu = 0.0$

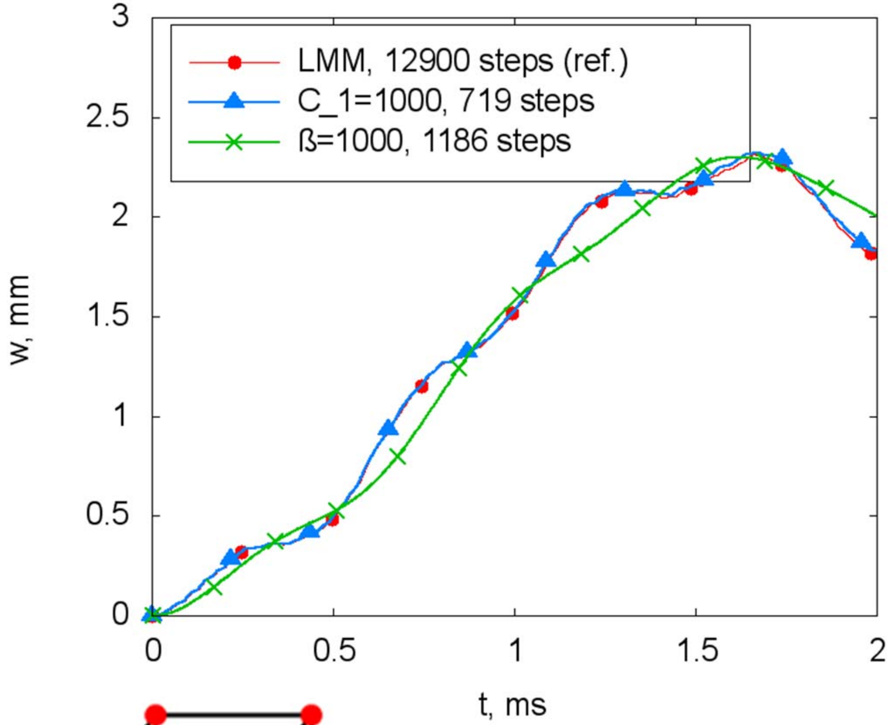
$\rho = 7800$ kg/m^3

$n_x = 50$

$n_y = 1$

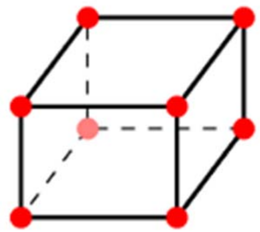
$n_z = 3$

$t_{end} = 2$ ms



Shape functions for velocity:

$$\mathbf{\Psi}_{3D} = \begin{bmatrix} 1 & 0 & 0 & -Y^h & Z^h & 0 \\ 0 & 1 & 0 & X^h & 0 & -Z^h \\ 0 & 0 & 1 & 0 & -X^h & Y^h \end{bmatrix}$$



variational approach for selective mass scaling is more accurate here: low order modes are dominant

OLOVSSON ET AL. (2005) SELECTIVE MASS SCALING FOR EXPLICIT FINITE ELEMENT ANALYSES, IJNME 63

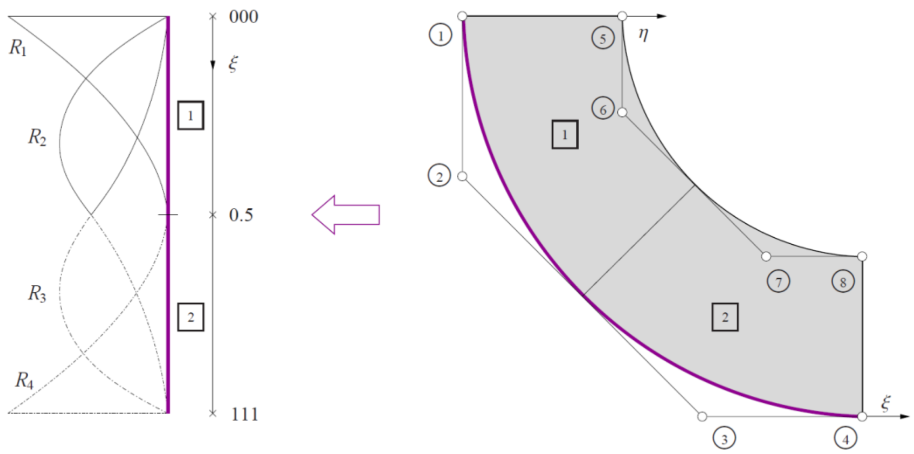
isogeometric analysis

benefits

smooth geometry, high continuity
direct transfer from CAD to CAE(?)
potential for new paradigm in FEM
→ there is more to come!

limitations

typical problems of classic FEM (locking)
many technical issues to be solved
before reaching maturity for industrial applications



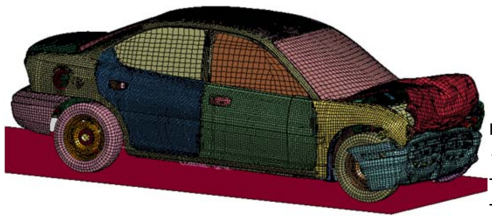
variational selective mass scaling

benefits

speed-up due to larger critical time step
outperforms algebraic mass scaling
consistent, general framework
→ there is more to come!

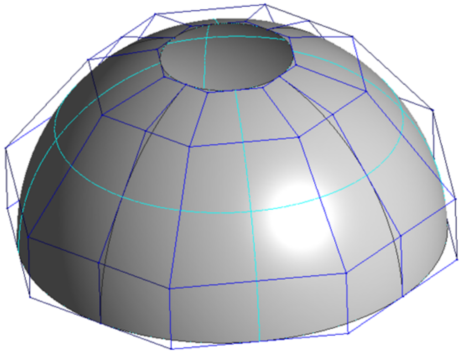
limitations

extra expense and loss of accuracy
application-dependent



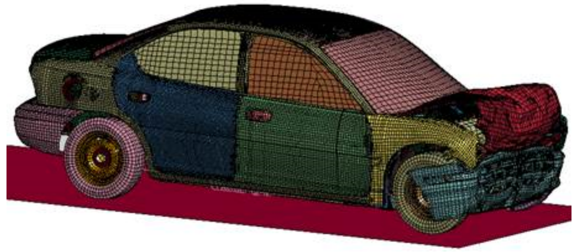
o n nds)		%CPU	Clock(seconds)	%Clock
	+01	0.10	2.3306E+01	0.09
	+04	70.75	1.7468E+04	70.80
Binary databases	6.2000E+01	0.25	6.9039E+01	0.28
ASCII database	2.4000E+01	0.10	1.8748E+01	0.08
Contact algorithm	7.0830E+03	28.71	7.0711E+03	28.66
Rigid bodies	2.4000E+01	0.10	2.3625E+01	0.10
T o t a l s	2.4674E+04	100.00	2.4674E+04	100.00

```
Problem time      =   4.0001E+01
Problem cycle     =    63493
Total CPU time    =    24674 seconds (   6 hours 51 minutes  )
```



LS-DYNA Forum 2014

7 October 2014
Bamberg



On two Recent Advances in Computational Mechanics

Isogeometric Analysis of Shells and Variational Mass Scaling

Manfred Bischoff,
Ralph Echter, Bastian Oesterle, Martina Matzen, Ekkehard Ramm,
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